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NOTE.—The closing dates herein published, are final except when names of prospective discussers are registered for special extension of time

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

RAINFALL INTENSITIES AND FREQUENCIES

By A. J. Schafmayer[1], M. Am. Soc. C. E., and B. E. Grant[2], Esq.

Synopsis

An investigation of the relation of frequency to rainfall intensity, by the statistical method, forms the basis of this paper. All the available records of excessive rainfall published by the United States Weather Bureau for nineteen cities were first examined and those exceeding certain rates were tabulated and plotted on semi-logarithmic paper. The graphs were straight lines of marked regularity in their arrangement. Then, the data from ten cities of the original nineteen were tabulated and plotted, thus using about one-half the quantity of data first used. A similar regularity and consistency in the curves were found. Finally, the data from fourteen rain-gages in the Chicago (Ill.) District were used for finding curves and formulas for the use of the City of Chicago. The formulas for intensity are rectangular hyperbolas and are plotted on hyperbolic paper as straight lines. The study is limited to excessive storms having durations of 120 min, and less.

Introduction

Rainfall intensity is involved in the design of storm sewers, as the rate of run-off is generally a function of the intensity. Adequate knowledge of intensities only becomes available with the accumulation of long-time records of automatic recording rain-gages. As the automatic gage has been in general use for only a few decades, formulas for intensity derived in the early part of the Twentieth Century were based on scanty information and should be revised.

The Board of Local Improvements of Chicago, a department of the Municipal Government which designs and builds most of the City's sewers, has been using a formula for rainfall intensity of the exponential type which was adopted in 1915. It was based on the rather scant information about the rainfall of Chicago available at that time. (The Sanitary District of Chicago,

Note.—Discussion on this paper will be closed in May, 1937, *Proceedings*.

[1] Engr., Board of Local Impvts., City of Chicago, Chicago, Ill.

[2] Div. Engr., Board of Local Impvts., City of Chicago, Chicago, Ill. Mr. Grant died on October 30, 1936.

a separate and independent municipality, builds and maintains intercepting sewers which carry the dry-weather flow of the sewers of Chicago and other cities to treatment plants. Its sewers, not being owned or controlled by the City of Chicago, are not classed herein as city sewers.)

The multiplication of gages in recent years and the corresponding increase in the available data have led to a more complete study of rainfall intensities and frequencies by the Board than had previously been made, and to the search for formulas that would be the basis for more rational and economical design.

The study included the determination of a method for using data from a number of gages and an investigation of the validity of the station-year method for utilizing such data. Some of the results of that study are presented herewith in an abridged form.

The hyperbolic type of formula for intensity of rainfall, originated about forty years ago by A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., has been found to fit the data more consistently than the exponential type. This study includes only storms of 2-hr duration and less, and the formulas are not intended for longer durations. However, this time is sufficient for the majority of cases in the design of city sewers.

In order to obtain a sufficient quantity of data on excessive storms, the records of a number of rain-gages in different cities are combined. These records give consistent and characteristic curves which afford a basis for judging the sufficiency of the lesser number of records which are available in a single city.

The rainfall at any given point results from the operation of a number of unknown laws. It is classified, therefore, as a chance event. If rainfalls of certain intensities and frequencies do follow a law within limits, the question arises as to how many data are needed to determine such law. The record from a single rain-gage for 100 yr is much more valuable than the record from the same gage for 25 yr, or even for 50 yr.

The tipping-bucket gage of the U. S. Weather Bureau which gives short-time records of excessive storms has not been in use long enough to give the desired quantity of data, and, therefore, some method must be used which will give reasonable results with scanty data.

The method of approach used in this paper accepts and classifies the available authentic data of a region or an area having somewhat similar rainfall characteristics and thus accumulates a number of observations great enough to indicate a rule or a law.

Definitions of Terms

The following definitions, abbreviations, and letter symbols apply thoroughout this paper:

"Intensity", i, is the rate of rainfall, in inches per hour.

"Duration", t, is the elapsed time, in minutes, for the excessive rainfall.

"Quantity of precipitation", d_a, is the total rainfall, in inches, for the time given.

"Frequency", F, is the average time, in years, between occurrences.

"Station-years" for a number of stations is the sum of the years of record for all stations in the group.

"Excessive rainfalls" are those equal to or greater than the smallest quantities used in the tabulations. These quantities are:

Rainfall, in inches	Duration, in minutes	Rainfall, in inches	Duration, in minutes
0.35	5	1.20	60
0.50	10	1.30	100
0.65	15	1.40	120
0.90	30		

"Chance event", is one in which the determining factors are unknown or are so complex that the result can be predicted only as an average, if at all.

Deficiencies in Data

When one attempts to find an expression for intensity of rainfall for any given frequency for Chicago, several difficulties appear:

(*a*) The record of excessive storms is a comparatively short one.

(*b*) The number of gages having an available record is too small.

(*c*) The published records give the quantity of rainfall for successive 5-min intervals, and it is only by chance that these may correspond to maximum quantities for short durations.

(*d*) The type of gage in use does not make a correct record of high intensities of short duration.

(*e*) The published summaries of the U. S. Weather Bureau give excessive rates of rainfall for durations of 2 hr and less, only.

(*f*) A single rain-gage gives a record of the precipitation at one point only, with no indication of the variation in any direction. It merely takes a sample, 1 ft wide, out of a storm that may have a width of 50 000 ft.

(*g*) A large number of storms might pass over a single gage before it would make a record of the maximum intensity of any one storm.

(*h*) It seems that no one has ever installed enough gages to show a reasonably correct picture of an excessive rain storm. In the Chicago District (see Fig. 1 and Table 1) with an area of about 250 sq miles, there are now (1937) fourteen rain-gages of the tipping-bucket type. The distance between them is still too great to give satisfactory storm patterns, although an effort has been made to use the available information.

These deficiencies probably explain inconsistencies in the results of rainfall studies and justify efforts to increase the statistical material applicable to a specific problem so as to reduce the percentage of probable errors. The addition of more gages multiplies the data each year and increases the points of application; for instance, in one Chicago Sewer District, comprising 4 500 acres, three gages were installed in 1933 to furnish data for a special study of the District as the entire run-off passes through a pumping station equipped

with recording Venturi meters. It is hoped that some definite relation may be established between rainfall intensity, impervious area, rate of run-off, total run-off, and time.

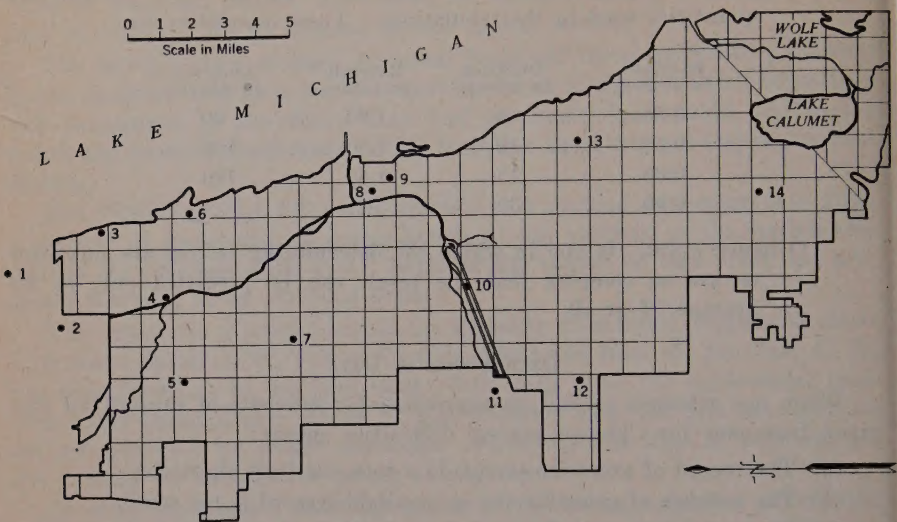

FIG. 1.—LOCATION OF RAIN-GAGES, CHICAGO, ILL. (SEE TABLE 1).

TABLE 1.—LOCATION OF RAIN-GAGES IN THE CHICAGO, ILLINOIS, DISTRICT

Rain-Gage No. (see Fig. 1)	Location	Address
	(a) UNITED STATES WEATHER BUREAU	
13........	University of Chicago..................	Fifty-ninth Street and Ellis Avenue
8........	United States Court House.............	Adams and Clark Streets
12........	Municipal Airport.....................	5932 South Cicero Avenue
	(b) CITY OF CHICAGO, ILLINOIS	
5........	Mayfair Pumping Station..............	4850 Wilson Avenue
7........	Springfield Avenue Pumping Station.....	1747 North Springfield Avenue
14........	Roseland Pumping Station.............	351 West One-Hundred and Fourth Street
6........	Lake View Pumping Station............	742 Montrose Avenue
	(c) SANITARY DISTRICT OF CHICAGO	
1........	Evanston Pumping Station.............	1457 Elmwood Avenue, Evanston
2........	North Side Treatment Works...........	Howard Street and McCormick Road
9........	Standard Oil Building..................	910 South Michigan Avenue
10........	Electric Sub-station.....................	Thirty-first Street and Western Avenue
11........	West Side Treatment Works............	Pershing Road and Fifty-second Avenue
3........	Loyola University......................	Sheridan Road and Loyola Avenue
4........	Winnetka (Discontinued September 27, 1933) (Now (1937) North Branch Pumping Station)	560 Orchard Lane, Winnetka Lawrence Avenue and Franciso Avenue

RELATION OF MAXIMUM PRECIPITATION TO TABULATED (PUBLISHED) QUANTITIES FOR 5, 10, AND 15 MINUTES

The tabulation of excessive rainfall published by the U. S. Weather Bureau shows the quantities for successive 5-min intervals, beginning with the excessive rate. The maximum may occur partly in one and partly in another of

the 5-min intervals. When this occurs the published records do not show this maximum. An examination was made of all the original rain-gage charts of the excessive rainfall records for the Federal Building, in Chicago, for the 10-yr period, 1919–1928, to find the relation of maximum precipitation for selected 5, 10, and 15-min periods to the tabulated precipitation as published. The results of this study are shown in Column (1), Table 2. Because "excessive rainfall" as used in this discussion is greater than that used by the Weather Bureau, the number of storms shown is materially smaller. The results of a similar study of these storms are shown in Column (2), Table 2. In somewhat more than one-half the storms the maximum quantities were

TABLE 2.—Relation of Maximum Precipitation to Tabulated Precipitation as Published

Description	Successive Interval, in Minutes, Beginning with the Excessive Rate of Rainfall					
	5		10		15	
	Storms classified as excessive by U. S. Weather Bureau (1)	Storms in which the maximum rainfall rate exceeded minimum used herein (2)	Storms classified as excessive by U. S. Weather Bureau (1)	Storms in which the maximum rainfall rate exceeded minimum used herein (2)	Storms classified as excessive by U. S. Weather Bureau (1)	Storms in which the maximum rainfall rate exceeded minimum used herein (2)
Number of storms tabulated.....	65	12	62	17	52	18
Number of storms in which the maximum rate exceeded tabulated rates..................	42	6	33	11	29	11
Percentage excess of maximum rainfall over tabulated rainfall.	9.9	3.2	3.3	2.1	2.3	2.0

greater than those shown in the official tabulations for the given intervals. The increase averaged from 2 to 3% for that part of the record used for the curves.

This record of one gage for ten years may be taken as an indication of the failure of the tabulated records to show maximum rainfall for short durations, but the data are too scanty to give general averages and no correction factor has been applied in this study.

New Rule of the United States Weather Bureau for Tabulation of Excessive Rainfall

The U. S. Weather Bureau adopted rules, in March, 1934, relating to the tabulation of excessive precipitation which are expressed by two formulas, one for the Southern States and one for the Northern States. Table 3, con-

TABLE 3.—Limits at Which Precipitation May Be Considered Excessive

Description	Duration of Storm, in Minutes										
	5	10	15	20	25	30	35	40	45	50	60
Depth of precipitation, in inches..........	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.80

tained in a set of instructions issued by the Weather Bureau, explains these rules It is based upon the formula:

$$d_a = t + 20 \quad \text{.............................} \quad (1)$$

in which d_a is the accumulated depth, in hundredths of inches; and t is the time, in minutes. For stations in the Southern States (including North Carolina, South Carolina, Georgia, Florida, Alabama, Mississippi, Tennessee, Arkansas, Louisiana, Texas, and Oklahoma), where brief, heavy showers are comparatively frequent, the excessive rate, for the purpose of tabulation, is based upon the formula:

$$d_a = 2t + 30 \quad \text{...........................} \quad (2)$$

For a rain lasting 5 min, for example, the total depth in Table 3 would be 0.40 in.; for $t = 30$ min, $d_a = 0.90$ in.; and, for $t = 60$ min, $d_a = 1.50$ in.

The U. S. Weather Bureau rule provides that the accumulated depths are to be tabulated when the fall equals or exceeds the rates determined by Equations (1) and (2) and that all tabulations must show the accumulations for 60, 80, 100, and 120 min, even where the excessive rate does not continue for such periods.

The rule for the Northern District applies to Chicago. However, the storms rated as excessive in this paper are in accordance with the definition previously given. The data published under the new rules are more complete and permit of more accurate analysis inasmuch as the precipitation for excessive storms is given for intervals as long as 120 min, although the excessive rate may not have continued for that length of time. This was not done in records previous to 1933, and any precipitation occurring within the 120-min period, but subsequent to the expiration of the excessive rate, was not shown.

Station-Years and Frequencies

The data on excessive rainfall from a single station are very scanty because the length of the record giving intensities for short durations extends over only a short term of years, and if storms of long-time frequencies occur in the record, it is difficult, if not impossible, to determine their frequencies. The best way to overcome this defect seems to be to combine the records of several stations and thus to obtain sufficient data to treat them statistically. The idea of combining records of different stations in hydraulic studies is not a new one. It was used by the late W. E. Fuller, M. Am. Soc. C. E.[3], by A. F. Meyer, M. Am. Soc. C. E.[4], and by the late Allen Hazen, M. Am. Soc. C. E.[5] The data in Meyer's compilation of excessive precipitation ended with the year 1914. Since then the total data available have been more than doubled by the publications of the U. S. Weather Bureau.

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[3] *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 564.
[4] "Hydrology", by A. F. Meyer, John Wiley & Sons, 1928.
[5] "Flood Flows", by Allen Hazen, John Wiley & Sons, 1930, p. 90.

If there are 10 stations with a 30-yr record for each station, there are 300 station-yr. A record that occurs three times in the entire 30 yr has a probable frequency of 100 yr for any one station, or the chance of such a storm occurring at a given station in any year is 1 in 100. In the same manner a record that occurs thirty times has an average frequency of 10 yr, and a record that occurs sixty times has a frequency of 5 yr. If the distribution were uniform and if each station showed that a certain record occurred six times in the 30 yr, the 5-yr frequency would be apparent, and the record of one station, in so far as this record was concerned, would be as good as the record of ten stations, but the distribution is not uniform and an average frequency is sought. The use of several rain-gages distributed throughout the area considered is a reasonable method of accumulating the needed data if the record extends over a considerable number of years.

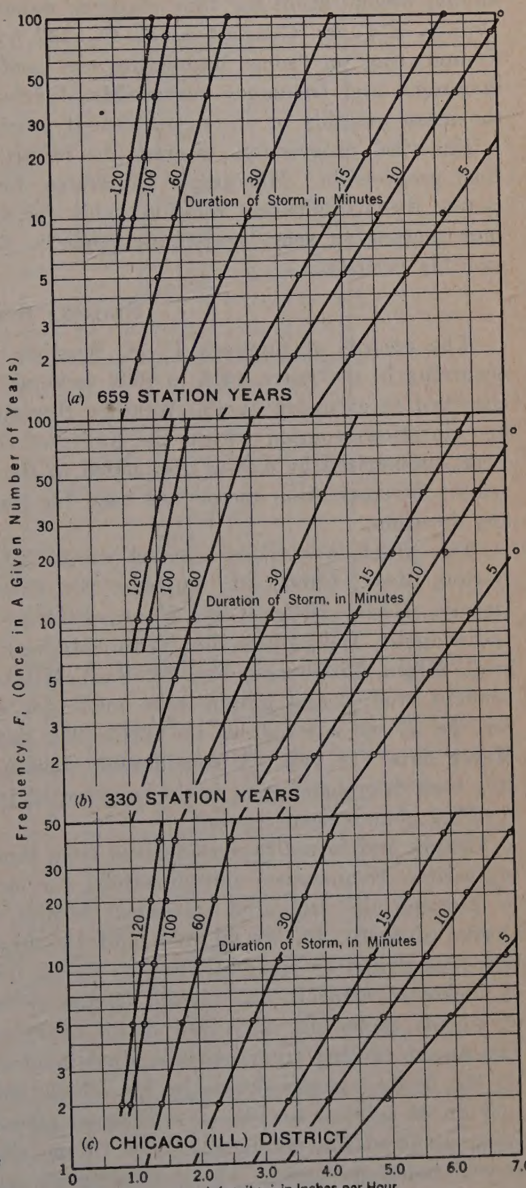

FIG. 2.—FREQUENCY-INTENSITY GRAPHS, FROM EQUATIONS OF THE FORM, $i = \frac{d_a}{t + b}$ (SEE TABLE 5).

Intensity Curves on Semi-Logarithmic Paper

The curves from formulas for intensity of the compound interest law type when plotted on semi-logarithmic paper are straight lines, and, therefore, the intensities for any given duration have a constant difference for frequencies that vary in a geometric ratio, such as 5, 10, 20 and 40 yr.

The lines that depart most from the vertical show the greatest difference in intensities for any given frequencies; hence, in Fig. 2, the 5-min line

shows the greatest differences and the 120-min line the least, as would be expected.

In 1930, C. S. Jarvis, M. Am. Soc. C. E., presented[6] a chart of frequency curves on semi-logarithmic paper, which shows the minimum and maximum annual precipitation for four stations, namely: Padua, Italy, 202 yr; Paris, France, 214 yr; Charleston, S. C., 137 yr; and, Boston, Mass., 111 yr. Commenting on "some interesting and useful relations" regarding periodic maximum and frequency curves, Mr. Jarvis calls attention to the fact that the mean monthly or the mean annual precipitation seems to form a simple arithmetical progression, whereas the related time intervals form a geometrical progression. His graph[6] illustrates, for the yearly precipitation, "how nearly the straight-line relation holds for frequency curves when they are thus plotted on semi-logarithmic paper." Similar results are apparent for monthly periods.

Stations Studied

The records of nineteen U. S. Weather Bureau Stations for all storms occurring in the years, 1915 to 1932 (several thousand in number), that were classified as excessive by the Weather Bureau, were examined for intensities and durations coming within the limits of this study. The records selected were summarized by classes and added to the Meyer summary for the earlier years. No correction factor was used for the precipitation corresponding to any duration.

The nineteen stations selected were: Albany, N. Y.; Asheville, N. C., Boston, Mass., Cairo, Ill., Chicago, Ill., Cincinnati, Ohio, Cleveland, Ohio, Detroit, Mich., Dodge City, Kans., Elkins, W. Va., Grand Haven, Mich., Indianapolis, Ind., Knoxville, Tenn., Madison, Wis., Memphis, Tenn., Moorhead, Minn., Pittsburgh, Pa., St. Paul, Minn., and Yankton, S. Dak. Independent studies and graphs were made for each of these nineteen stations for the 37 yr covered by the 1915–1932 data (18 yr) combined with the Meyer data (19 yr). A considerable degree of similarity was found with only such discrepancies as might be expected from data of such limited nature covering chance occurrences.

Graphs and formulas were derived from these data, covering the quantities, intensities, frequencies, and durations for excessive storms. It was found, by plotting the tabulated statistical results on semi-logarithmic paper for storms of 5, 10, 15, 30, 60, 100, and 120-min duration, that the data were best represented by straight lines, as shown in Fig. 3(a). The precipitation, in inches of rainfall, was plotted on the arithmetical scale and the number of storms exceeding the given depth of rainfall on the logarithmic scale. The means of class intervals were used in plotting the curves. For example, on the 30-min graph shown in Fig. 3(a), 353 storms were plotted against 1.05 in. of rainfall for the class interval extending from 1 in. to 1.10 in., it being assumed that as many storms in the class interval exceeded the mean as were below the mean. The class intervals vary for storms of different durations, but are the same for each duration in all these studies. With this

[6] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), Fig. 3, p. 400.

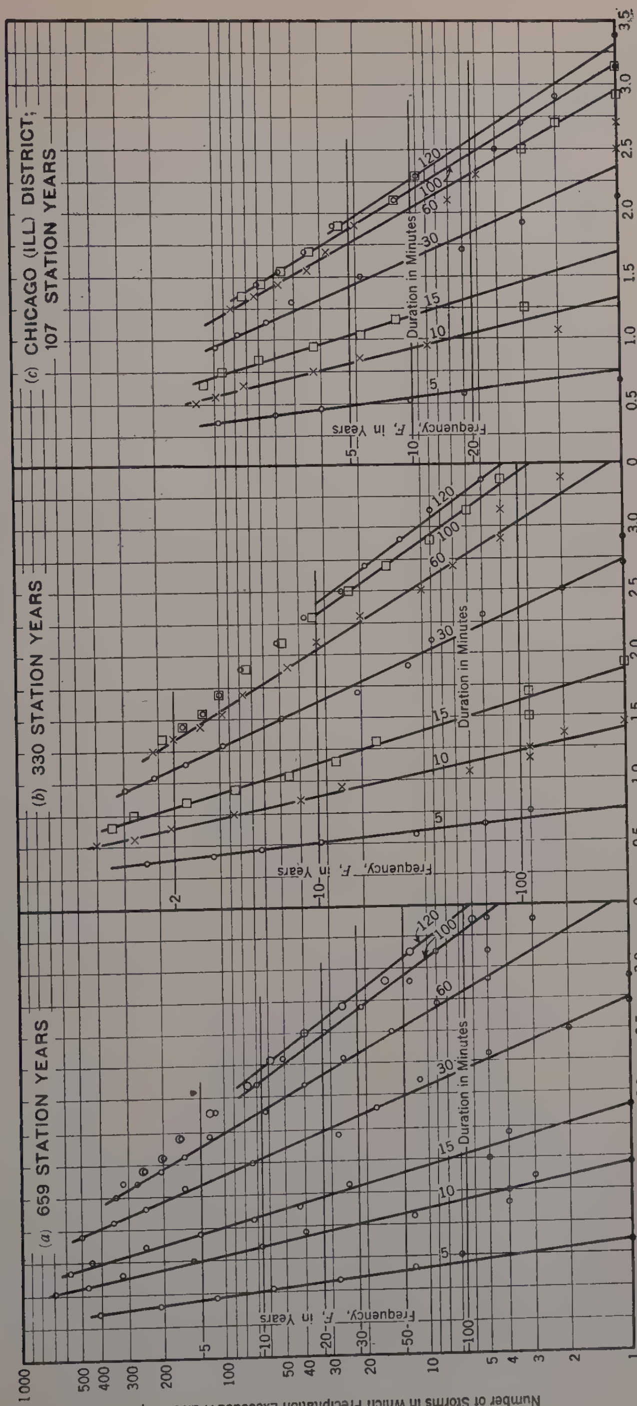

FIG. 3.—GRAPHS OF STATISTICAL DATA.

TABLE 4.—SUMMARY OF RECORDS OF STORMS AT TEN STATIONS, 1900 TO 1932

Precipitation, in inches (1)	Knoxville, Tenn. (2)	Memphis, Tenn. (3)	Indianapolis, Ind. (4)	Cincinnati, Ohio (5)	Detroit, Mich. (6)	Chicago, Ill. (7)	St. Paul, Minn. (8)	Cairo, Egypt (9)	Yankton, S. Dak. (10)	Dodge City, Iowa (11)	Total (12)	Summary (13)
(*a*) PRECIPITATION IN 5 MINUTES												
0.35 to 0.39	14	12	20	11	11	14	10	9	10	8	119	227
0.40 to 0.44	4	4	6	3	8	3	4	3	7	4	46	108
0.45 to 0.49	4	1	4	3	4	3	6	1	1	3	30	62
0.50 to 0.54	1	0	3	4	1	3	2	1	2	4	21	32
0.55 to 0.59	0	1	0	0	0	2	2	0	0	1	6	11
0.60 to 0.69	0	1	0	0	0	0	0	1	0	0	2	5
0.70 to 0.79	0	1	0	1	1	0	0	0	0	0	3	3
0.80 to 0.89	0	0	0	0	0	0	0	0	0	0	0	0
Total......	23	20	33	22	25	25	24	15	20	20	227	.....
(*b*) PRECIPITATION IN 10 MINUTES												
0.50 to 0.54	12	17	16	14	13	7	12	22	17	6	136	396
0.55 to 0.59	9	13	13	10	3	10	6	7	12	3	86	260
0.60 to 0.69	9	5	16	8	10	7	7	8	11	8	89	174
0.70 to 0.79	4	3	4	2	7	5	6	5	4	5	45	85
0.80 to 0.89	2	0	4	2	1	4	1	0	1	0	15	40
0.90 to 0.99	2	1	2	2	1	2	2	2	1	4	19	25
1.00 to 1.09	0	1	0	0	0	0	0	0	0	2	3	6
1.10 to 1.19	0	0	0	0	0	0	0	0	0	0	0	3
1.20 to 1.29	0	1	0	0	0	0	0	0	0	0	1	3
1.30 to 1.39	0	0	0	0	1	0	0	0	0	0	1	2
1.40 to 1.49	0	0	0	1	0	0	0	0	0	0	1	1
Total......	38	41	55	39	36	35	34	44	46	28	396	.....
(*c*) PRECIPITATION IN 15 MINUTES												
0.65 to 0.69	5	13	11	5	6	6	5	13	7	4	75	334
0.70 to 0.79	14	14	15	9	8	11	7	14	12	10	114	259
0.80 to 0.89	4	3	14	5	4	6	9	5	7	5	62	145
0.90 to 0.99	3	4	2	2	8	5	1	5	3	5	38	83
1.00 to 1.09	4	1	5	2	1	1	2	1	1	0	18	45
1.10 to 1.19	0	0	2	0	0	3	0	0	1	4	10	27
1.20 to 1.39	3	0	1	2	0	1	2	2	1	2	14	17
1.40 to 1.59	0	0	0	0	0	0	0	0	0	0	0	3
1.60 to 1.79	0	1	0	1	0	0	0	0	0	0	2	3
1.80 to 1.99	0	0	0	0	1	0	0	0	0	0	1	1
Total......	33	36	50	26	28	33	26	40	32	30	334	.....
(*d*) PRECIPITATION IN 30 MINUTES												
0.90 to 0.99	6	13	10	11	7	4	4	9	13	8	85	295
1.00 to 1.09	5	10	8	5	5	8	4	8	6	3	62	210
1.10 to 1.19	2	9	7	1	4	6	4	9	5	5	52	148
1.20 to 1.39	4	2	5	0	6	7	5	6	7	5	47	96
1.40 to 1.59	5	1	2	4	2	2	3	1	2	6	28	49
1.60 to 1.79	1	0	1	1	0	1	0	3	2	0	9	21
1.80 to 1.99	1	0	1	0	0	0	0	0	0	1	3	12
2.00 to 2.19	1	0	0	0	0	1	2	0	0	0	4	9
2.20 to 2.39	0	0	0	1	1	0	0	0	0	1	3	5
2.40 to 2.59	0	0	0	1	0	0	0	0	0	0	1	2
2.60 to 2.79	0	0	0	0	0	0	0	0	0	0	0	1
2.80 to 2.99	0	1	0	0	0	0	0	0	0	0	1	1
Total......	25	36	34	24	25	29	22	36	35	29	295	.....

method of plotting, double points occur on certain lines as on the 10-min line in Fig. 3(*b*). The list of excessive storms shown in Table 4 indicates a total of three storms for each of the intervals, 1.10 to 1.19 in. and 1.20 to 1.29 in. The statistical results for the 330 station-yr study, given in Table 4, is typical. The respective class intervals are shown in Column (1) and the number of storms shown in Column (13) were the values plotted for each class interval. The deviation of individual points from a straight line was a matter of only a few hundredths of an inch of rainfall, except in

TABLE 4.—(*Continued*)

Precipitation, in inches (1)	Knoxville, Tenn (2)	Memphis, Tenn (3)	Indianapolis, Ind (4)	Cincinnati, Ohio (5)	Detroit, Mich (6)	Chicago, Ill (7)	St Paul, Minn (8)	Cairo, Egypt (9)	Yankton, S Dak (10)	Dodge City, Iowa (11)	Total (12)	Summary (13)
(*e*) PRECIPITATION IN 60 MINUTES												
1.20 to 1.29	2	6	3	5	3	5	5	6	5	2	42	209
1.30 to 1.39	7	4	2	2	4	6	3	4	3	6	41	167
1.40 to 1.49	3	5	3	2	2	4	3	1	4	3	30	126
1.50 to 1.59	1	3	4	1	0	1	2	6	2	0	20	96
1.60 to 1.79	3	5	3	2	3	2	1	3	6	3	31	76
1.80 to 1.99	2	1	0	1	0	2	0	1	3	2	12	45
2.00 to 2.19	0	0	3	2	1	0	1	2	1	3	13	33
2.20 to 2.39	1	0	0	1	1	2	2	0	1	2	10	20
2.40 to 2.59	2	0	0	0	0	0	1	0	0	0	3	10
2.60 to 2.79	0	0	1	1	0	0	1	0	0	0	3	7
2.80 to 2.99	0	0	0	0	0	0	0	0	0	0	0	4
3.00 to 3.24	0	0	0	0	1	0	0	1	0	0	2	4
3.25 to 3.49	0	1	0	0	0	0	0	0	0	1	2	2
Total......	21	25	19	17	15	22	19	24	25	22	209	.....
(*f*) PRECIPITATION IN 100 MINUTES												
1.30 to 1.39	7	3	2	1	4	6	3	5	2	8	41	192
1.40 to 1.49	3	2	2	3	2	4	4	3	3	4	30	151
1.50 to 1.59	1	3	5	1	0	2	2	3	2	1	20	121
1.60 to 1.79	2	4	2	4	1	2	2	3	4	3	27	101
1.80 to 1.99	2	5	2	0	3	2	2	3	5	1	25	74
2.00 to 2.19	0	1	2	1	1	1	1	4	2	1	14	49
2.20 to 2.39	0	2	2	2	0	2	1	0	0	3	12	35
2.40 to 2.59	2	1	0	1	0	0	1	1	1	1	8	23
2.60 to 2.79	1	0	0	1	2	0	1	0	0	1	6	15
2.80 to 2.99	1	0	1	0	0	0	1	0	0	0	3	9
3.00 to 3.24	0	0	0	0	0	0	0	1	1	0	2	6
3.25 to 3.49	0	0	0	0	0	0	0	0	0	0	0	4
3.50 to 3.74	0	0	0	0	0	0	0	1	0	0	1	4
3.75 to 3.99	0	0	0	0	1	0	0	0	0	0	1	3
4.00 to 4.49	0	0	0	0	0	0	0	0	0	0	0	2
4.50 to 4.99	0	1	0	0	0	0	0	0	0	1	2	2
Total......	19	22	18	14	14	19	18	24	20	24	192	.....
(*g*) PRECIPITATION IN 120 MINUTES												
1.40 to 1.49	3	2	2	3	2	4	4	3	3	4	30	152
1.50 to 1.59	1	3	5	1	0	3	2	3	2	1	21	122
1.60 to 1.79	2	2	2	2	1	2	2	3	4	3	23	101
1.80 to 1.99	2	4	2	2	3	2	2	3	5	1	26	78
2.00 to 2.19	0	3	2	1	1	1	1	3	1	1	14	52
2.20 to 2.39	0	2	2	2	0	2	1	1	0	3	13	38
2.40 to 2.59	1	1	0	1	0	0	1	0	1	1	6	25
2.60 to 2.79	1	0	0	1	2	0	1	1	0	0	6	19
2.80 to 2.99	1	1	1	0	0	0	0	0	1	0	4	13
3.00 to 3.24	0	0	0	0	0	0	1	1	1	1	4	9
3.25 to 3.49	1	0	0	0	0	0	0	0	0	0	1	5
3.50 to 3.74	0	0	0	0	0	0	0	1	0	0	1	4
3.75 to 3.99	0	0	0	0	1	0	0	0	0	0	1	3
4.00 to 4.49	0	0	0	0	0	0	0	0	0	0	0	2
4.50 to 4.99	0	1	0	0	0	0	0	0	0	1	2	2
Total......	12	19	16	13	10	14	15	19	18	16	152	.....

certain cases where the indicated frequency exceeded 100 yr, which is to be expected, as the correct frequency of rare storms is always a matter of uncertainty in a short record.

Some frequency lines are also shown on Fig. 3(*a*). The 10-yr frequency is represented by the horizontal line through the point, 65.9, which represents one storm in 10 station-yr. A frequency-quantity graph was derived directly from Fig. 3(*a*) by plotting the values of the depths for the 100-yr frequency line and the depths for the 5-yr frequency line, and drawing straight lines

between the points plotted. In other words, such a graph would be similar to Fig. 3(a), except that the curves are inverted, and the logarithmic scale then gives the frequency for any number of years from 1 to 100.

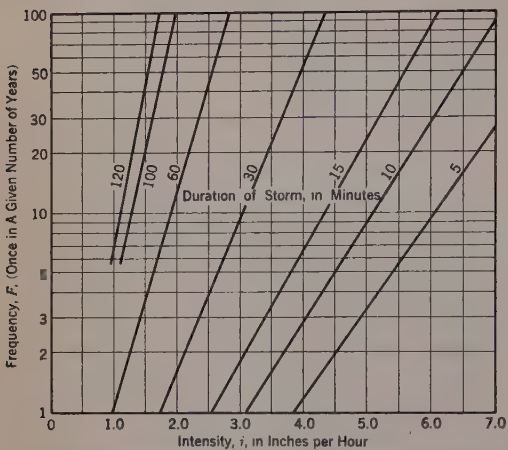

FIG. 4.—FREQUENCY-INTENSITY CURVES OF STATISTICAL DATA; 65 STATION-YEARS.

Fig. 4 gives the relation between frequency and intensity for the nineteen stations. These graphs were obtained directly from the aforementioned frequency-quantity graph by converting the quantities for the various durations into intensities, or rates per hour, and by plotting the frequency on the logarithmic scale. This chart shows frequencies up to 100 yr, but as the data cover 659 station-yr, it might be reasonable to extend the graphs to an even higher limit.

Although the relation between durations and depths of rainfall for any frequency appears to follow a law as expressed by logarithmic formulas, it was found that the hyperbolic type of formula represented by,

$$i = \frac{A}{t + b} \quad \text{.........................(3)}$$

was the best form for showing the relation between intensity and duration for storms not exceeding 120 min of a given frequency. (In Equation (3), A and b are abstract constants, as explained subsequently under the heading "Constants in the Formulas for Intensity".) Formulas of this type were derived from the 659-station-yr data. These formulas were then plotted, as shown by dots in Fig. 2(a), and furnish a basis of comparison with the graphs as derived from the statistical data as shown on Fig 4. The points

TABLE 5.—VARIATIONS OF CONSTANTS IN THE FORM OF EQUATION (3)

Item No.	Formulas for intensity of rainfall, i, used in diagrams	Frequency, in Years: 2		5		10		20		40		80	
		Numerator, A	b	Numerator, A	b	Numerator, A	b	Numerator, A	b	Numerator, A	b	Numerator, A	b
1	2(a).........	97	16	126	18	152	20	177	21	202	22	227	23
2	2(b) and 5....	102	16	138	19	166	21	193	22	222	24	251	25
3	2(c)..........	108	17	137	18	157	18	181	19	202	19	.....	..

as plotted from these formulas give a straight line for each duration, indicating a remarkable consistency in the formulas. These graphs give values very close to those shown by the statistical graphs, except in the case of the

5-min line, and probably the values given by the formula are more nearly correct for this duration than the values given by the statistics. The curves of Fig. 2(a) are variations of Equation (3), as defined by Item No. 1, Table 5.

Historical

The hyperbolic type of formula has been used since 1891, and the credit for it belongs to Professor Talbot. In reply to an inquiry concerning the history of it, Professor Talbot wrote to the writer on July 31, 1933, as follows:

"In 1891 when I was making a study of rates of rainfall as affecting desirable size of stormwater sewers the only reference found in engineering literature was an article by Professor Nipher of St. Louis in the *American Engineer,* May 8, 1885, which gave a formula for intensity having a constant in the numerator and the time in the denominator $\left(i = \dfrac{360}{t}\right)$. Nipher's data were limited to St. Louis.

"The printed reports of the U. S. Weather Bureau gave the rainfall for certain time periods, but little or no attention was given to the rate or intensity of the rainfall as such. After plotting the rates against time for all the data available in the published records of the U. S. Weather Bureau and for different sections of the Country and for the larger cities, my studies indicate that a formula of the form of a constant in the numerator and the time plus a constant in the denominator would best envelop the plotted points for 'rare rainfalls' and another of the same form those for what were called 'ordinary rainfalls.' The studies further brought out that the maximum rates shown by the accumulation of data were not greatly different for the several sections of the country considered (the maximum rates, not their frequency), a conclusion quite different from the views then commonly held.

"The studies were published as 'Maximum Rates of Rainfall' in *The Technograph,* No. 6, 1891–92, University of Illinois. This article was republished in whole or in part in engineering journals and has been abstracted or referred to through the years in discussions and publications here and abroad. For thirty years hardly a year went by without seeing a reference to it in print.

"So far as the form of the formula is concerned, it was devised by me, and I have seen nothing since that indicated a prior use by another. Mr. Kuichling told me later that he had used a straight-line formula in 1889. Mr. Rudolph Hering once showed me the rainfall plots he had made for New York City, but after reading my article he had not gone further in the study of the subject.

"In the years intervening since the publication of 'Maximum Rates of Rainfall' there has been a great and valuable accumulation of rainfall statistics. It would not be strange if now a new constant or even a new form of formula would better fit the data. Naturally I am pleased to learn that the form of the formula devised by me so many years ago is useful in expressing the intensity of rainfall for Chicago."

Some of the hyperbolic formulas in common use are of the Talbot type. Formulas of this type that are used in other cities have been studied in connection with the frequency-intensity graphs, Figs. 2(a) and 4, and also on hyperbolic paper with varying results. Considerable variation in fre-

quency has also been found for formulas of the exponential type such as,

$$i = \frac{A}{(t + b)^n} \quad \text{.........................(4)}$$

Thus far, it has not been found, in any of these investigations, that a formula of this type would give a constant frequency for all durations between 5 and 120 min. Rectangular hyperbolas fit the data better than any other curves and the Talbot type of formula is the simplest expression found for them for a given frequency.

Limitations of Formula for Intensity

Although the hyperbolic type of formula appears to be singularly adapted to express the intensity for a given frequency, for excessive storms of durations not exceeding 120 min, it is, from its fundamental structure, not applicable for storms of long duration, as shown by the following analysis. The formula,

$$i = \frac{222}{t + 24} \quad \text{.........................(5)}$$

gives the intensity or rate per hour for a 40-yr frequency curve. If it is multiplied by $\frac{t}{60}$, the result will be the precipitation, d, for the time, t. Then:

$$d_a = \frac{222}{t + 24} \times \frac{t}{60} = \frac{3.7\,t}{t + 24} \quad \text{.....................(6)}$$

The factor, $\frac{t}{t + 24}$, will always be less than unity for any value of t and, therefore, the value of d_a cannot exceed 3.7 in. for any time. When t becomes large, the change in d_a is small for successive changes in t, as shown by the following tabulation:

Duration, t, in minutes	Precipitation, d_a, in inches
120	3.08
600	3.56
1 440	3.64
2 880	3.67

As the rainfall in 10 hr, or 24 hr (600 min, or 1 440 min) may greatly exceed the corresponding values shown for d_a, Equation (6) is evidently not applicable for the longer durations.

A formula for depth of rainfall was derived as a function of log-time which fits closely the data used in the hyperbolic formula (Equation (3)) from $t = 10$ to $t = 120$ min and does not have the defect of reaching a maximum.

In the case of the 40-yr frequency, it was:

$$d_a = 1.89 \log t - 0.80 \quad \text{.....................(7)}$$

The hyperbolic form is simpler for computation and is the one preferred when the time extends only to 120 min.

Curves for Ten Selected Stations

After analyzing the data for the entire nineteen stations results were examined to discover the ten stations having characteristics similar to Chicago conditions. This was done for the purpose of securing data that corresponded most closely to the Chicago conditions, in order to amplify limited records available locally. The ten stations were: Cairo, Ill., Chicago, Ill., Cincinnati, Ohio, Detroit, Mich., Dodge City, Kans., Indianapolis, Ind., Knoxville, Tenn., Memphis, Tenn., St. Paul, Minn., and Yankton, S. Dak.

After selecting these stations an independent study of the records of the U. S. Weather Bureau for the 33 yr from 1900 to 1932, inclusive, was made and the excessive storms were tabulated as shown in Table 4. Similar tables, which are not shown, were prepared for the studies for 659 station-yr, and for Chicago. Graphs were made similar to the graphs prepared for the 659 station-yr in the preliminary studies.

It was found that the use of the data for storms of low intensity added little to the value or the accuracy of the graphs. In general, Table 4 includes only storms having intensities equal to or greater than those indicated in the definition of excessive storms. These intensities include all frequencies likely to be of interest to the designer of storm sewers for the reason that sewers should be designed to provide for storms of greater intensity than those likely to occur once a year or oftener.

Fig. 3(b) shows the statistical data for ten stations for the years, 1900 to 1932, inclusive, or a total of 330 station-yr plotted on semi-logarithmic paper, with the depth of rainfall shown on the arithmetical scale and the number of storms on the logarithmic scale Fig. 3(b) corresponds, for the ten stations, with Fig 3(a) showing the nineteen stations, and is similar in the relative position of the graphs. More than 1 800 excessive storms were tabulated and studied to make the chart for ten stations.

Fig. 2(b) shows frequency-intensity graphs for 330 station-yr plotted from the hyperbolic formulas which were derived from the statistical data. For the ten stations, Fig. 2(b) is comparable with Fig. 2(a) for the nineteen stations. With the exception of the 5-min and 10-min lines, the graphs fit the original data so nearly that they are practically coincident. From the formulas (Equation (3), and Items Nos. 1 and 2 in Table 5), the 5-min line is probably more nearly correct than the original data, as it gives larger values. The equation for the 60-min curve on this chart may be written in two forms:

$$i = 1.03 + 1.00 \log F \qquad (8)$$

or,

$$F = 0.0933\ e^{2.3i} \qquad (9)$$

in which e is the Naperian base. The curves of Fig. 2(b) are variations of Equation (3), as defined by Item No. 2, Table 5.

Derivation of a Formula for the 60-Minute Graph on the Frequency Intensity Chart for 330 Station-Years (See Fig. 2(*b*))

Any curve that appears as a straight line on semi-logarithmic paper has an equation of the form:

$$y = k\ e^{rx} \qquad (10)$$

in which k is a constant, and r is the rate of increase. Substituting F, the frequency, and i, the intensity, for y and x, Equation (10) becomes:

$$F = k\ e^{ri} \qquad (11)$$

Taking values of i from the curve for the 60-min storm (Fig. 2(*b*)) when $F = 100$ and $F = 1$ and substituting them in Equation (11): $100 = k\ e^{3.03\ r}$; and, $1 = k\ e^{1.03\ r}$. Then, converting to the logarithmic form:

For $F = 100$:

$$\log 100 = \log k + 3.03\ r \log e \qquad (12a)$$

for $F = 1$:

$$\log 1 = \log k + 1.03\ r \log e \qquad (12b)$$

and, by subtraction,

$$2 = 2.00\ r \log e \qquad (13)$$

Finally, $r = \dfrac{1}{0.4343} = 2.30$; and, by substituting this value in Equations (12) and solving simultaneously: $\log k = -1.03 = 8.97 - 10$; $k = 0.0933$; and, substituting in Equation (11):

$$F = 0.0933\ e^{2.3i} \qquad (14)$$

Equation (14) may be reduced to the formula for intensity, as follows:

$$\log F = \log 0.0933 + 2.3\ i \log e \qquad (15)$$

and since $2.3 \log e = 1$,

$$\log F = \log 0.0933 + i \qquad (16)$$

and,

$$i = \log F - \log 0.0933 = \log F + 1.03 \qquad (17)$$

which may also be obtained by inspection of Fig. 2(*b*). The equation for any other line on Fig. 2(*b*) can be obtained in a similar manner:

For $t = 120$,

$$i = 0.57 + 0.61 \log F \qquad (18a)$$

for $t = 100$,

$$i = 0.67 + 0.70 \log F \qquad (18b)$$

for $t = 60$,

$$i = 1.03 + 1.00 \log F \qquad (18c)$$

for $t = 30$,

$$i = 1.80 + 1.46 \log F \qquad (18d)$$

for $t = 15$,

$$i = 2.73 + 1.87 \log F \qquad (18e)$$

for $t = 10$,

$$i = 3.32 + 2.03 \log F \qquad (18f)$$

and, for $t = 5$,

$$i = 4.18 + 2.25 \log F \qquad (18g)$$

The constant is the value of the intensity for the 1-yr frequency and the coefficient of $\log F$ is the 10-yr intensity minus the constant. As the logarithm of 10 is 1, it is apparent that the value of the 10-yr intensity is the sum of the constant and the coefficient.

The Chicago Curves

The methods of studying the nineteen stations and, later, the ten stations were then applied to the data of the U. S. Weather Bureau for the Chicago Station. Following the procedure previously described for the nineteen stations, a chart was prepared with graphs showing storms from 5 to 60-min duration, based on the statistical data for a period of 33 yr, which were all the official data available for that station, for excessive storms of those durations.

Other rain-gages in the Chicago District furnished records that might be combined with the data of the Weather Bureau Station, to give charts based on nearly three times the statistics of the single station. Thirteen rain-gages of the automatic type furnished the precipitation data for the City of Chicago used in this study. The location of the gages is shown in Fig. 1 and Table 1. Of these, two are maintained by the U. S. Weather Bureau (three in 1937), four by the City, and seven by the Sanitary District of Chicago (six in 1937). The distribution of the gages, although not ideal, is still fairly good as may be seen by reference to the outline map of Chicago in Fig. 1.

Three gages were established in 1933. Five gages have records beginning in 1926. The records of excessive precipitation from two of the automatic gages of the Federal Government extend back to 1919 and 1900. It is quite possible in combining these records that too much emphasis is given to records of recent years, but there is no obvious method for balancing this defect except by adding new data as they occur. Seven years of observation with the present number of gages will about double the data.

The official station (for observation purposes) of the U. S. Weather Bureau, in Chicago, was on the United States Court House (formerly the Federal Building) previous to 1926, when the official station was moved. Rosenwald Hall and the adjoining campus at the University of Chicago then became the official station (for observation purposes), although it had been

in existence as an unofficial station of the Weather Bureau since 1916. The station on the U. S. Court House has been continued, but its records have not been published since 1925.

Curves Applying to the Chicago District

Fig. 3(*c*) shows the statistical data for the combined records of two Weather Bureau gages, and the several gages maintained by the City of Chicago and the Sanitary District of Chicago to the end of 1933, amounting to 107 station-yr. The longest record for one gage is for 34 yr. Fig. 2(*c*) shows frequency-intensity graphs for storms from 5 to 120-min duration, based on formulas derived from the foregoing statistics. The plotting of the formulas in this case shows the same consistency of the points on the lines and the relative position of the graphs that was shown in Figs. 2(*a*) and 2(*b*). The curves of Fig. 2(*c*) are variations of Equation (3), as defined by Item No. 3, Table 5.

Semi-Hyperbolic Paper

The Talbot formula, Equation (3), is a special form of the general equation $xy = k$, representing a rectangular hyperbola. Since the value of x varies as a hyperbola, the locus of the equation should be a straight line when plotted on semi-hyperbolic paper, that is paper graduated in one dimension on an arithmetical scale and in the other dimension on a hyperbolic scale.

As an investigation failed to disclose any hyperbolic paper of suitable size on the market, paper of this type was prepared. For the purpose a form of semi-hyperbolic paper of letter-sheet size was prepared with the hyperbolic scale covering 10 in. and the arithmetical scale, 7 in. The hyperbolic scale is derived from the formula for a rectangular hyperbola. In this case the base line equals 10 in., and the divisions from the right-hand margin are equal to ten times the reciprocals of the numbers from 1 to 10. For example: The line, 5, is ten times $\frac{1}{5} = 2$ in. from the margin; the line, 2, is ten times $\frac{1}{2} = 5$ in. from the margin; the value of the line at the left margin is 1.0; and the value of the line at the right margin is infinity.

Equations of the form, $xy = k$, which includes equations for intensity of rainfall, such as $i = \dfrac{126}{t + 18}$, appear as straight lines on this paper. It has a range for $i = 1$ to $i = 10$; and a range for $t = 0$ to $t = 120$.

Intensity Curves on Semi-Hyperbolic Paper

Fig. 5 shows, on semi-hyperbolic paper, the intensity-duration relationship for storms varying in frequency from 2 to 80 yr as determined by the formulas derived from the studies for the ten stations previously mentioned, including 330 station-yr (see Table 5). The frequencies shown, for $F = 5$, 10, 20, 40, and 80 yr, are in a geometrical progression; the 2-yr frequency is an exception. The graph for the 10-yr frequency is midway between the graphs for the 5-yr and 20-yr frequencies, and the graph for any

term in the geometrical progression lies midway between the graphs of the adjacent terms. The application of this relationship for interpolating intermediate frequencies and for deriving formulas will be discussed subsequently.

The intensity curves on hyperbolic paper are best adapted for use in preparing sewer designs. The range of the charts is sufficient to include all except the larger sewer projects. Curves are given for five or six different frequencies and for durations of as much as $t = 120$ min with 2-min intervals. Graphs for other frequencies than those shown, can easily be added if needed.

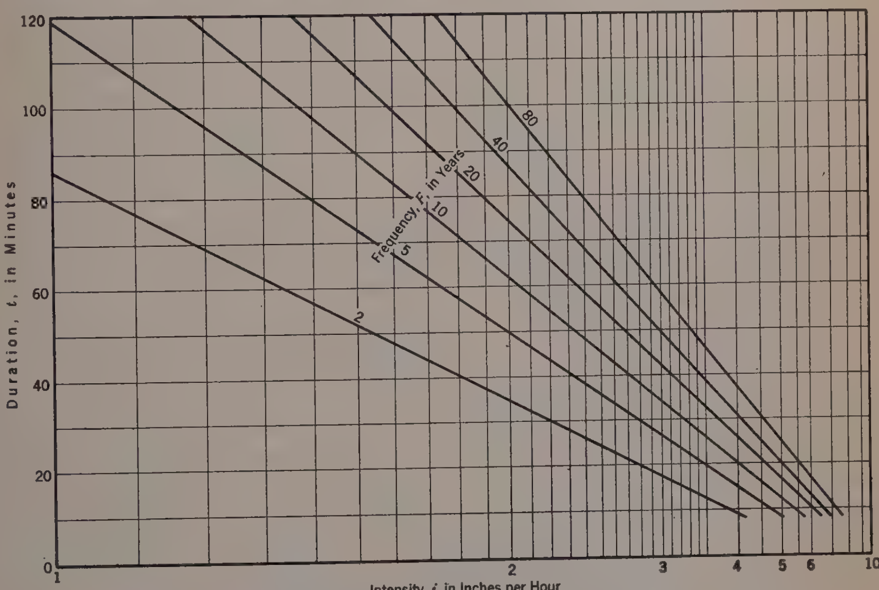

FIG. 5.—INTENSITY-DURATION GRAPHS; 330 STATION-YEARS (SEE TABLE 5, ITEM NO. 2).

A brief examination of these charts reveals the great importance of the time element. A change of 10 min in the estimate of the concentration period, as from 50 to 40 min, may be the equivalent of changing from a 5-yr frequency to a 10-yr frequency; for example, the intensity for a 5-yr frequency is 2 in. for 50 min, and 2.33 in. for 40 min, the latter being the same as the intensity for a 10-yr frequency for 50 min.

THE CONSTANTS IN THE FORMULAS FOR INTENSITY

The constants in the formulas for intensity were found originally by trial to make the formulas fit the values in the statistical data. This is easily done with the slide-rule. They may also be found by solving Equation (3), using the values for i and t that are obtained from the graphs of the statistical data (Fig 3). After the formulas were determined for the various frequencies, the results were verified, and the accuracy and consistency of the formulas and data were examined by plotting the constants in the numerators and denominators on semi-logarithmic paper.

Fig. 6 shows the constants used in the intensity formulas applying to the data for nineteen Weather Bureau stations. The values for the constants in the numerators lie on a straight line with one exception and the values

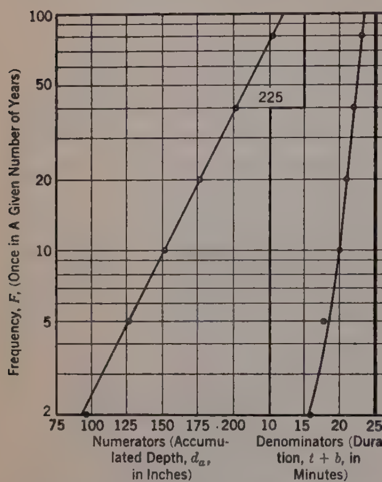

FIG. 6.—CONSTANTS IN INTENSITY FORMULAS, 659 STATION-YEARS.

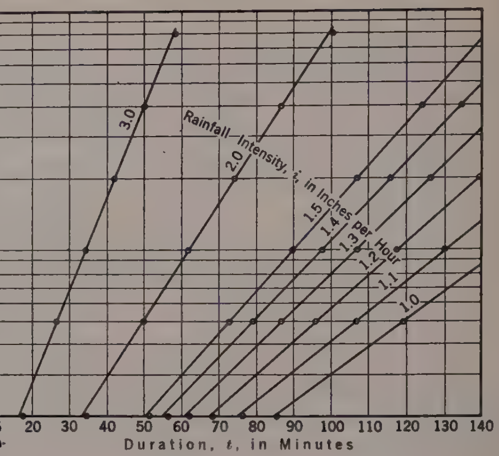

FIG. 7.—FREQUENCY DURATION GRAPHS FOR THE INTERPOLATION OF FREQUENCIES.

used in the denominators are close to a line with very slight curvature. All the constants in these formulas are whole numbers, as it seemed that the introduction of fractions would be a needless refinement. By the use of the graphs the approximate values of the contents in the formulas for any desired frequency from 2 to 100 yr may be obtained, and formulas for the intensity for any desired frequencies within the range of the chart may be written. This gives a simple method for computing two values and drawing any intermediate frequency line on the hyperbolic chart.

GRAPHICAL INTERPOLATION OF FREQUENCY LINES

A graphic method for interpolating frequencies on a hyperbolic chart of rainfall intensities for the ten stations, is shown in Fig. 7, in which intensity lines are drawn with frequency and time as the co-ordinates. In other words, it is a frequency-duration chart for various intensities. The points were found by solving for time in the intensity formulas for the several frequencies with assumed intensities from 1 in. per hr to 3 in. per hr. These points, for frequencies of 5 yr, or more, when plotted on the semi-logarithmic paper give a straight line for each intensity; for example, if it is desired to interpolate on the hyperbolic chart, Fig. 5, a line giving the intensities for a frequency of 15 yr, two values should be found for the time for that frequency, using different intensities. For an intensity, $i = 3$, Fig. 7 gives the time, $t = 39$ min, and for $i = 1.4$, $t = 108$ min. This permits the plotting of two points on the 15-yr line on the hyperbolic chart (Fig. 5) and the drawing of a straight line through these points, which will give the intensities for any time up to 120 min.

The formula adopted in 1914–15 for use in Chicago was of the form of Equation (4):

$$i = \frac{28}{t^{0.7}} \qquad (19)$$

This curve resembles a hyperbola, but it is not a true hyperbola. The intensities computed from it have been plotted on the frequency intensity chart for the Chicago District and are shown by the curved line in Fig. 8. This

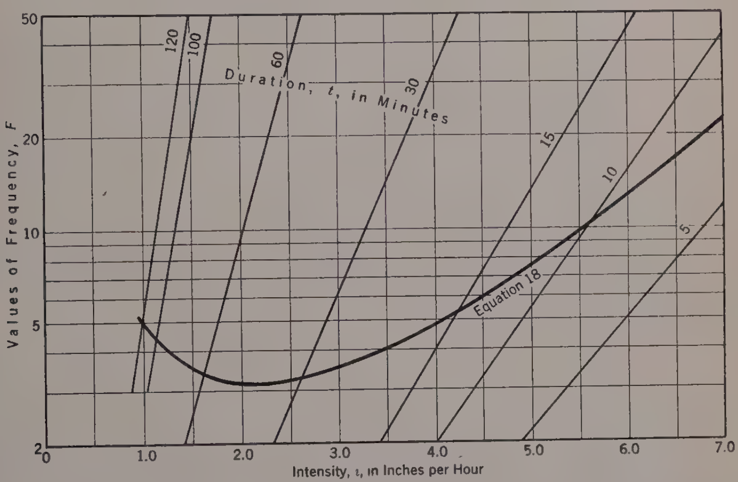

FIG. 8.—CURVES SHOWING VARIABLE FREQUENCY OF OLD FORMULA.

graph indicates that the frequencies represented by Equation (19) vary from about 3 yr to more than 20 yr. Before these studies were made it was thought that the freqnency corresponding to the intensities obtained by Equation (19) was approximately 5 yr, but such is the fact only for durations of 15 and 120 min, and it is not true for storms of other durations.

CONSTANT RATIO FOR FREQUENCIES WHEN INTENSITIES INCREASE BY A FIXED SUM

A characteristic of any formula which plots as a straight line on semilogarithmic paper, is that adding a fixed sum to the intensity will multiply the frequency by a fixed sum. In other words, successive additions of a fixed sum to the intensity will correspond to frequencies with a constant ratio. As an example, consider the following quantities picked from the 60-min frequency curve for 330 station-yr:

Intensity, in inches	Frequency, in years	Intensity, in inches	Frequency, in years
1.25	1.6	2.00	8.9
1.50	2.85	2.25	16.0
1.75	5.0	2.50	28.5

In each case, the ratio between successive frequencies is consistently 1.78.

Ratio of Depths of Rainfall (and Intensities) from Curves for 330 Station-Years

Table 6 shows depths of rainfall for storms of 1, 10, and 100-yr frequency for durations of 5, 10, 15, 30, and 60 min, taken from graphs; it also shows the computed ratio of the depth corresponding to a frequency higher than the ratio for the next lower frequency. The precipitations for various durations are shown, but the intensities will have the same ratios.

TABLE 6.—Comparison of Frequency Studies

Frequency, F, in years	Duration, t									
	5		10		15		30		60	
	d_a, in inches	Ratio	d_a, in inches	Ratio	d_a, in inches	Ratio	d_a, in inches	Ratio	d_a, in inches	Ratio
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	0.34		0.54		0.67		0.91		1.04	
		1.50		1.63		1.70		1.79		1.96
10	0.51		0.88		1.14		1.63		2.04	
		1.33		1.40		1.42		1.44		1.49
100	0.68		1.23		1.62		2.35		3.04	

An equation such as,

$$i = \frac{C F^n}{(t + 7)^{0.7}} \qquad (20)$$

will not fit the data in Table 6 because F, with a constant exponent, indicates a constant ratio of the intensities for all durations when the frequencies are in a geometrical progression. The ratio of intensities actually increases with the duration and decreases with the frequency in Table 6.

Interpolation of Frequency Curves by Geometric Mean

Charts on hyperbolic paper show that if the frequency lines are in a geometrical progression they will intercept a given intensity line with equal intervals (time) between them; for example, the 10-yr frequency line in Fig. 5 crosses the 2-in. intensity line half way between the 5-yr and the 20-yr frequency lines. Similarly, the line midway between the 5-yr and 10-yr lines will have a frequency of 7.07 yr, which is the square root of 5 × 10, and the line midway between the 10 and 20-yr lines will have a frequency of 14.14 yr, which is the square root of 10 × 20. A geometrical progression has the form,

$$x = a + ar + ar^2 + ar^3 + ar^4 \ldots . + ar^{n-1} \qquad (21)$$

in which a = the first term; r = the ratio; and n = the number of terms. The ratio of the last term to the first term $\left(\frac{\text{Last term}}{\text{First term}}\right)$ is:

$$\frac{a\, r^{n-1}}{a} = r^{n-1} \qquad (22)$$

Then, for the interpolation between $F = 5$ and 10, or $F = 10$ and 20:

$$\log r = \frac{1}{n - 1} \log 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

When the first and last terms are given, all the intermediate terms may be found.

Solving a 5-term series for three intermediate frequencies between 5 and 10 yr gives lines for $F = 5.95$, 7.07, and 8.41 yr, equally spaced. These values may be found by taking square roots, but when the exact position is required for an integral frequency, it is generally necessary to find an exponent by logarithms

The integral numbers, 5, 6, 7, 8, 9, and 10, apparently, are terms in an arithmetical progression, but they are also approximate terms in a geometric progression that has a large number of terms (say, 1 001). Applying this method it is found that the spacing for the frequencies, $F = 6$, 7, 8, and 9 will be, respectively, 26.3%, 48.5%, 67.8%, and 84.8% of the distance between the 5-yr and the 10-yr frequencies. Intermediate integral frequencies between 10 and 20 will have the same proportionate spacing.

Difference in Time When Frequencies Are in Geometrical Progression

If the several formulas for intensity used in Fig. 5 (see Table 5, Item No. 2) that were derived from the chart showing the data for 330 station-yr are solved for the time with the same assumed intensity, it is found that the differences in time or in duration are constants for each intensity when the frequencies are in a geometrical progression. This is also true within narrow limits for the corresponding formulas for intensity for the Chicago District. This characteristic indicates a possible method for drawing additional curves on a chart.

TABLE 7.—Differences in Duration Time, Δt

Frequency, F	Intensity of Rainfall, in Inches per Hour							
	1		2		3		4	
	Time, t, in minutes	Difference, Δt	Time, t, in minutes	Difference, Δt	Time, t, in minutes	Difference, Δt	Time, t, in minutes	Difference, Δt
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
5	108		45		24		13½	
		24		11		6⅔		4½
10	132		56		30⅔		18	
		24		11½		7⅓		5¼
20	156		67½		38		23¼	
		24		11½		7⅓		5¼
40	180		79		45		28½	
		24		11½		7⅓		5¼
80	204		90½		52⅔		33¾	

Table 7 gives values of Δt from the formulas for 659 station-yr for frequencies of 5, 10, 20, 40, and 80 yr (see Table 5, Item No. 1). From inspection of Table 7 it appears that,

$$\Delta t = \frac{25 - i}{i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

when frequencies form a geometrical progression. If $i = 2$ in. per hr, $\Delta t = \frac{25-2}{2} = \frac{23}{2} = 11\frac{1}{2}$; and, if $i = 4$ in. per hr, $\Delta t = \frac{25-4}{4} = \frac{21}{4} = 5\frac{1}{4}$. The differences are constant for a given intensity when the frequencies vary as in a geometrical progression.

Conclusions

The following conclusions are indicated by the foregoing arguments:

(1) The statistical method which combines station-years affords a means for studying rainfall data which gives reasonably regular and consistent results.

(2) Semi-logarithmic and hyperbolic papers are effective tools in studies of rainfall.

(3) For the shorter durations, the inlet time is an important factor in the time of concentration, as suggested in the comment on Fig. 5. Studies to find its probable mean value are desirable.

(4) Formulas such as Equation (7) indicate that when the durations are in a geometrical series the differences in rainfall values are a constant. This suggests a method that may be useful in a study of storms exceeding 120 min. Fig. 5 indicates that when the frequencies are in a geometrical series the differences in durations for a given intensity are constant. These relations would seem to be more than a coincidence and may be worthy of more extended examination.

(5) For the 5-yr and the 10-yr frequencies a close degree of similarity is shown for intensity values by the curves for the Chicago District and those for the 330 station-yr, except for 5-min and 10-min durations.

(6) The frequency-intensity curves give the probable intensity of rainfall for certain durations for a single gage. The average intensity over a large area, of course, is less than the maximum intensity. The ratio of this average to the maximum intensity is as yet unknown, and is one of the factors needing further investigation. One of the methods suggested for overcoming this difficulty in sewer design is the use of a frequency which varies in some inverse ratio with the duration or time of concentration. The frequencies that should be used in sewer design, although theoretically a matter of economics, are actually a matter of judgment based on experience.

(7) A study should be made of the relation of average intensities to maximum intensities of rainfall on small areas, such as may be used in sewer design, say, to 10 000 acres. This would require a considerable number of gages spaced in a regular pattern so as to permit the drawing of isohyetals and the computation of average depths of rainfall. It is believed that the frequency graphs give values for areas that are comparable with the size of sewer districts, and that the maximum size of such areas may be between 1 000 and 10 000 acres.

(8) The values obtained from the charts developed for the Chicago District have greater statistical foundation and appear to be more reliable than

any curves previously used by the Board of Local Improvements, City of Chicago. These charts are now (1937) being used by the engineers of the Board.

Acknowledgments

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PAPERS

FLOW CHARACTERISTICS IN ELBOW DRAFT-TUBES[1]

By C. A. Mockmore,[2] M. Am. Soc. C. E.

Synopsis

The purpose of this study was to investigate the flow characteristics in the elbow draft-tube, placing special emphasis on the bent part of the tube. The first part of the study consisted of designing and building several different pipe bends and testing them with a Pitot tube for filamental velocities and pressures. Measurements for loss of head were made for various rates of flow to determine which shape of bend offered the least resistance to the flowing water.

The second part of the study involved the design, construction, and testing of several model draft-tubes, patterned after the type of those at the Bonneville Dam, on the Columbia River, in Oregon. The model draft-tubes, as well as the experimental pipe bends, were made of pyralin, a transparent material through which it was possible to take photographs of the phenomena. Motion pictures were made for all conditions of flow in the pipe bends and the draft-tubes.

The tests indicated that the pipe bend which was flattened in the direction of the plane of the bend offered less resistance to flow than any of the other bends, regardless of cross-sectional area. It also appeared that the distance between the inside and outside walls at the outlet of the bend of an elbow draft-tube should be small compared to the throat diameter of the tube.

Introduction

Function of Draft-Tubes.—The draft-tube of a water turbine is an extension of the wheel passages, made so as to perform two principal functions:

Note.—Discussion on this paper will close in May, 1937, *Proceedings*.

[1] Based upon a thesis submitted to the State University of Iowa in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The data supporting this paper have been filed complete for reference in the Library of the State University of Iowa, Iowa City, Iowa, and in Engineering Societies Library, in New York, N. Y.

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(1) To reduce the velocity of the water in an efficient manner as it passes through the tube; and (2) to permit the turbine runner to be placed at a convenient distance above the tail-water.

It would be difficult to state which of these two functions is the more important because their relative values may change from one installation to another, depending upon the physical conditions of the plant. Whatever the relative values may be, the total head should not exceed about 25 ft, and this should vary inversely with the specific speed of the turbine, in order that the danger of cavitation may be lessened.

Historical Development of Draft-Tubes.—For those who may be interested in the historical development of draft-tubes, Appendix I will serve as a valuable check list of reading material[3]. In general, they may be divided into two classes: (*a*) Those having straight axes; and (*b*) those having curved axes. This paper presents in detail a study of the flow characteristics in quarter-turn draft-tubes. Draft-tubes with straight axes, mentioned only incidentally herein, have been studied further at the State University of Iowa by Andreas Luksch (6).

The most recent and most complete study of draft-tube literature the writer has been able to obtain was reported in April, 1935, by the National Bureau of Standards (16), of the Department of Commerce, at the request of the Tennessee Valley Authority. According to the report,

"The purpose of this investigation was to examine and report on all published literature on draft-tubes published in English and foreign languages during the past 20 years, to prepare an annotated bibliography of this material, and to make recommendations as to the need for further experiments on draft-tubes, and as to the direction which such experiments should take."

"The current design of draft-tubes," according to the report, "shows a persistent tendency to depart from the use of the spreading and symmetrical types." This is attributed to the better structural features of the elbow draft-tubes and to the improvement in their hydraulic efficiencies. As an example of the findings in improved efficiency for the elbow draft-tube, reference is made to the Lilla Edet (17) experiments in which a "special curved tube, shallow and very long, proved to be superior to the Moody (10) and White (9) tubes at full load performance."

A listing of the recent installations of elbow draft-tubes, such as that given by J. S. Ball (18), seems to vindicate the pioneer work of the late Gardner S. Williams, M. Am. Soc. C. E., who was among the earliest to claim that the elbow draft-tube may have as high efficiency as any other tube built. However, this is not to be construed to mean that the elbow draft-tube will ever entirely displace all draft-tubes having straight axes. No one type of tube could possibly be superior to all other types under all conditions of operation. It was in this connection that Dr. D. Thoma (19) stated: "It is firmly established today that a given draft-tube may be the best possible form for one runner, but not for another. Consequently, the runner and draft-tube should be investigated as a single system."

[3] In this paper numerals in parentheses, thus (10), refer to corresponding numbers in Appendix I.

Characteristic Parts of a Draft-Tube.—It appears logical that an elbow-type draft-tube may be considered as composed of three principal parts: (1) The vertical leg; (2) the bend; and (3) the lower leg, sometimes called the horizontal leg.

In the report (16) by the Bureau of Standards, a hypothetical form of draft-tube was assumed as made up of four principal parts for the purposes of discussion. The fourth part was added following the bend, and assumed as consisting of a section of the tube in which there was no increase in cross-sectional area. It appears that this part was added to take care of the suggestion of Viktor Kaplan (13) that in his experiments he found that the addition of such a section had increased the efficiency of the draft-tube. To the writer it appears that this is merely a part of the bend, or should be considered as such. Many designers of elbow draft-tubes believe that, in designing the bent part, the cross-sectional areas from section to section along the axis should increase in diminishing degree until near the end of the bend there is little or no increase in cross-sectional area from section to section. Then, as the lower leg is encountered, the flare may again be resumed.

Experiments with Pipe Bends

Notation.—The symbols used throughout this paper are defined where they first appear. An effort has been made to conform as nearly as practicable with 'Symbols for Hydraulics'[4], compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1929.

Theory of Induced Spirals in Pipe Bends.—The bend of the quarter-turn draft-tube is the part in which the greatest complications arise in the function of velocity reduction. In fact, complications are inherent with the flow of water in any channel where the filaments are forced to undergo a general change of direction. If there were stream-line flow of equal velocities at the entrance to a bend, and if friction could be eliminated, no complications of any great import would be encountered. With a whirl component at the entrance to a pipe bend, even without frictional resistance, complications would arise due to gyral action.

No pipe can be made in which frictional resistance to flow will not exist. Therefore, the filaments of flow near the walls of the pipe will be of less velocity than those near the center, and, as the water enters a bend, a spiral flow is induced automatically.

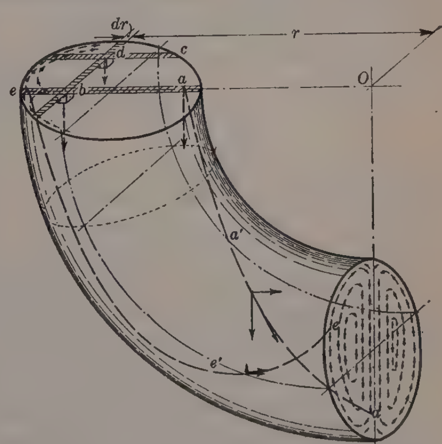


Fig. 1.—Induced Spiral Flow in a Pipe Bend.

[4] A.S.A.—Z10b—1929.

As an explanation of the cause of the spiral flow, let a small block of water, b, at a distance, r, from Point O (Fig. 1) have unit length and breadth, and a thickness, dr. Its volume then would be infinitesimally small as represented by the expression: $dQ = dr$; and its mass would be,

$$dm = \frac{w}{g} \, d\,Q = \frac{w}{g} \, dr \qquad (1)$$

in which w = the weight per unit volume; and g = the acceleration of gravity.

From mechanics it is known that the centrifugal force induced by the water moving with a velocity, V, being deflected (20) by a circular bend of radius, r, is:

$$dF = dm \, \frac{V^2}{r} = w \, V^2 \, \frac{dr}{r\,g} \qquad (2)$$

The area of the side of the small block is unity, since it is of unit length and breadth, and the intensity of pressure, increased on the left side of the small block caused by the centrifugal force, is:

$$dp = w \, V^2 \, \frac{dr}{r\,g} \qquad (3)$$

If the velocities of the filaments between Points a and b are considered equal, and if the inner radius is designated by r_i and the outer radius by r_o, the difference in pressure between these limits integrates into the following:

$$\Delta p = w \, \frac{V^2}{g} \int_{r_i}^{r_o} \frac{dr}{r} = \frac{w \, V^2}{g} \log \left(\frac{r_o}{r_i} \right) \qquad (4)$$

Equation (4) indicates that the difference in pressure caused by the centrifugal force is a function of the square of the velocity. It is similar to the expression for the impulse, or reaction of a jet of water:

$$F = \frac{W \, V}{g} \qquad (5)$$

in which, F = the force, in pounds; V = the velocity of the jet, in feet per second; and W = the flow of water, in pounds per second. The weight, in pounds per second, is then a function of velocity, so that the force, F, is a function of the square of the velocity. Thus, it appears logical that as the bend is encountered the pressure either should be increased at Point e (Fig. 1), or reduced at Point a by some amount, not necessarily exactly equal to that indicated by Equation (4), because the velocities along the traverse between Points a and e are not all of equal magnitude. These velocities will change as the pressures change in the flow around the bend, and perhaps changes will occur even in advance of the bend.

If all the filamental velocities of the water were equal at the entrance to the bend (a condition which could not be obtained easily without a very special apparatus) the pressure at Point d (Fig. 1) would be the same as

that at Point b; but, due to the lower velocities in the elementary strip, c-d, the pressure at Point d will be less than that at Point b. Therefore, a transfer of water from Point b toward Point d will tend to take place. Likewise, movement will tend to occur from Point a toward Point b, and a double spiral is induced. The two principal causes of this condition are: (1) The centrifugal force in the pipe bend; and (2) the existence of friction on the pipe walls so as to give higher velocities near the center of the pipe than near the walls. Under actual conditions the particles of water near Point e (Fig. 1), might progress in a path such as e-e'-e'', and another particle, such as at Point a, might progress in a path such as a-a'-a''.

An explanation of the induced spiral probably was offered first by Professor James Thomson (21), in 1876. Recently, tests on the flow of water around bends in pipes were conducted by D. L. Yarnell and the late F. A. Nagler (22), Members, Am. Soc. C. E., at the Iowa Institute of Hydraulic Research. These experiments were undertaken for the purpose of determining the laws governing the changes in pressure and velocity in different parts of the flowing stream, as the moving water undergoes the transition from motion along a straight path to motion around a curve, and then as it undergoes the opposite transition back to final straight-line motion. These experiments are especially valuable because of the numerous Pitot tube measurements of the velocity and pressure at the various sections of the pipe bends. Several different shapes of the pipe bends were used, and tests were made to determine the loss of head due to flow around each bend. The closing statement in the discussion of that paper is of particular interest to the designer of an elbow draft-tube: "Probably the most important finding in the investigation is that it is possible to have such conditions of flow that the loss of head may be very little, or unusually large, in the same bend for identical discharges", depending upon the velocity distribution at the entrance to the bend.

Several of the pipe bends reported in this paper were sent to the Iowa Institute of Hydraulic Research and were tested for flow characteristics. Only part of the results of these tests were described by Messrs. Yarnell and Nagler.

Design of Experimental Pipe Bends.—The general equation of continuity of flow of a fluid is:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V_x)}{\partial x} + \frac{\partial (\rho V_y)}{\partial y} + \frac{\partial (\rho V_z)}{\delta z} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

in which ρ = density of the fluid; t = element of time; V_x = component of velocity along the X-axis; V_y = component of velocity along the Y-axis; and V_z = component of velocity along the Z-axis.

The density of the water may be regarded as constant, and the equation of continuity becomes:

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 = \text{divergence} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

Equation (7) satisfies the specification for two-dimensional irrotational flow since the divergence is zero. If the motion is irrotational (23) the curl of

the velocity vector is zero, and the vector of which V_x, V_y and V_z are components is derivable from a potential.

For irrotational motion, the components of flow in the direction of the axes, X, Y, and Z, are: $V_x = -\frac{\partial \Psi}{\partial x}$; $V_y = -\frac{\partial \Psi}{\partial y}$; and $V_z = -\frac{\partial \Psi}{\partial z}$. The equation of continuity then becomes:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

which is the Laplace equation.

If this is taken as a special case of two-dimensional flow, say, in the X-Y plane, then,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The real part, ψ, of any function, $f(x + iy)$, of a complex variable satisfies the equation for the velocity potential, and, at the same time, the imaginary part gives the stream function. Assume that:

$$\Psi + i\Phi = (x + iy)^2 = x^2 - y^2 + 2i(xy) \ldots\ldots\ldots\ldots (10)$$

Therefore,

$$\Psi = x^2 - y^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

and,

$$\Phi = 2xy \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

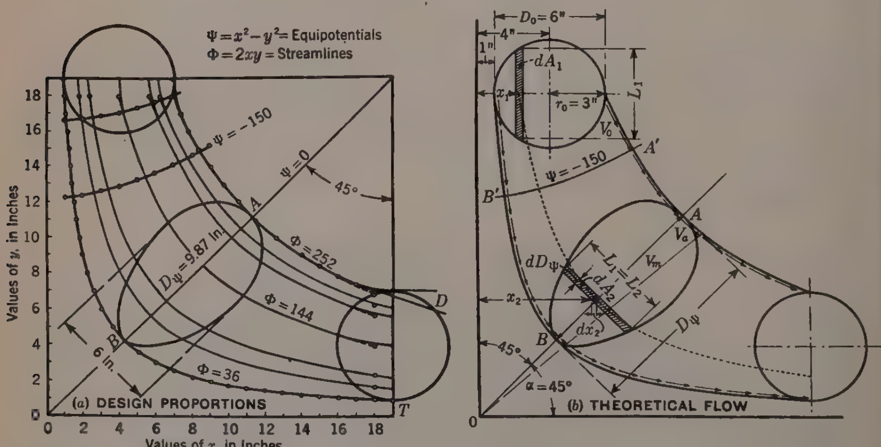


FIG. 2.—DESIGN OF PIPE BEND NO. 2.

Equations (11) and (12) satisfy the specifications for two-dimensional irrotational flow since the divergence is zero:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = \frac{\partial^2 (x^2 - y^2)}{\partial x^2} + \frac{\partial^2 (x^2 - y^2)}{\partial y^2} = 0 \quad \ldots\ldots\ldots (13)$$

To design a pipe bend from this relation and not to take friction into account, of course, would omit an essential factor in the flow of water in pipes, but it was deemed desirable to try it and observe the results. Accordingly, a bend suitable for a 6-in. pipe was designed, more or less by trial, until that shown in Fig. 2 was obtained. If the co-ordinates, in inches, of the beginning point, T, were used as (1, 12) instead of the (1, 18), the angle at Point D would become large. To use (0.5, 18) instead of (1, 18) made the width, A-B, rather large and, at the same time, gave a sharper curve near Point A. To avoid an abrupt angle at Point D, 1 in. was added to each of the x and y-distances to afford an opportunity to smooth out the hyperbolic curves to meet the parallel filamental flow from the 6-in. pipe at the entrance to the bend. The leg length, L, of the pipe thus became 15 in. as shown in Fig. 2.

The distance across the pipe from side-wall to side-wall, measured along the equi-potential lines, is expressed by the equation:

$$D_\Psi = \int \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{0.5} dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Since, $x^2 - y^2 = C =$ a constant; $2\,x\,dx - 2\,y\,dy = 0$; $\frac{dy}{dx} = \frac{x}{y}$; and, $\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{y^2}$:

$$D_\Psi = \int \left[1 + \left(\frac{x^2}{y^2}\right)^2\right]^{0.5} dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

When $x^2 - y^2 = \Psi = 0$ (which is the special case half-way around the bend), $D_\Psi = 9.87$ in.

The area of the cross-section of the pipe half-way around the bend, Fig. 2, may be expressed mathematically. Assuming, for example, $x\,y = 18 = C$, then $x_1 = \frac{C}{18}$. At the 45° section, $x = y$, so that, $(x_2)^2 = C = 18\,x_1$. Then, $x_2 = 3\,(2)^{0.5}\ (x_1)^{0.5}$; $dx_2 = \left(\frac{3}{2}\right) (2)^{0.5} \frac{dx_1}{x_1^{0.5}}$; $dD_\Psi = dx_2\,(2)^{0.5} = 3\,\frac{dx_1}{x_1^{0.5}}$; and, $dA_2 = L\,dD_\Psi = 3\,L\,\frac{dx_1}{x_1^{0.5}}$. Since the radius, r, of the entrance of the pipe is 3 in., the length, L, of the elemental area, dA, is:

$$L = 2\,[9 - (x_1 - 4)^2]^{0.5} \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

which is the same as the length of the corresponding elemental area for the section half-way around the bend. Then,

$$d\,A_2 = 3\,L\,\frac{dx_1}{x_1^{0.5}} = 6\int_1^7 \left(8 - x_1 - \frac{7}{x_1}\right)^{0.5} dx_1 \quad \ldots\ldots\ldots (17)$$

Equation (17) is an elliptic integral, which can be evaluated. By plotting the curve to scale and using a planimeter, the area was found to be 47.0 sq in.

The generating angle of a cone having an equivalent increase in area in the same length as this hyperbolic bend between entrance and vertex was computed and found to be 4° 20′, which is well below the usual allowable value of the angle of flare for draft-tubes. This bend hereafter will be referred to as Bend No. 2. Bend No. 1 is a standard 6-in. pipe bend of 90° having a center-line radius of 8 in.

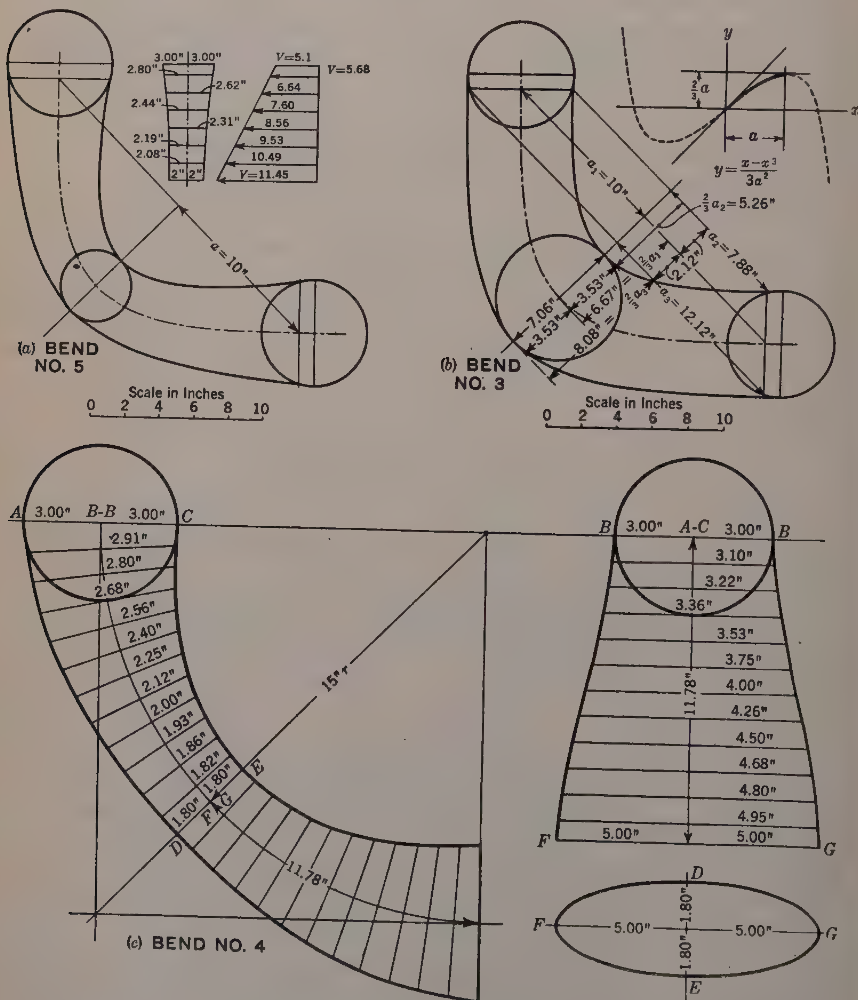


Fig. 3.—Design of Pipe Bends.

S. M. Woodward, M. Am. Soc. C. E., has suggested designing a bend with side-walls defined by the cubic equation,

$$y = x - \frac{x^3}{3\,a^2} \qquad (18)$$

This curve has a slope of 45° at the origin, and is horizontal at its maximum point where $x = a$, and $y = \frac{2a}{3}$. The radius of curvature is infinity at the origin, and is $0.5\,a$ at the maximum point. The part of the curve between the origin and the maximum point was used to make one-half the bend; and then it was turned over to make the other half of the bend, as shown in Fig. 3.

By assuming values of a in Equation (18), the side-walls of the bend were obtained. This bend would always bulge at the vertex. The final dimensions of the bend, as adopted for testing, had a diameter at the vertex of 7.06 in., an entrance diameter of 6 in., and a leg length of 15 in. This was obtained by taking values of a for the inside wall, the axis, and the outside wall, equal to 7.88, 10, and 12.12, in., respectively. By using these values of a the leg length would have become 14.14 in., but this was arbitrarily increased to 15 in., making the additional leg length have parallel side-walls as in a 6-in. cylinder. Hereafter, this pipe bend will be designated as Bend No. 3.

Professor Nagler suggested making a bend having all cross-sections elliptical and of equal area. The axis of the bend was a quadrant with a radius of 15 in. as shown in Fig. 3. The maximum width of section was arbitrarily fixed at 10 in., giving a maximum angle between axis and side-wall of about 10 degrees. It is apparent from Fig. 3 that the side-walls, *B-G* and *B-F*, are not straight lines, but are drawn in to form smooth curves. By assuming the distance, *F-G*, as 10 in., and maintaining an area at the vertex equivalent to that of a 6-in. circle, the ellipse had the dimension, *D-E*, of 3.6 in. Hereafter, this bend will be referred to as Bend No. 4.

Another bend (No. 5) was designed by using the same axis as in the case of Bend No. 3, and applying Equation (18) with $a = 10$ in. The side-walls were designed to constrict the flow at the vertex of the bend, giving an arbitrary minimum diameter of 4 in. The increase in velocity from entrance of bend to the mid-section was assumed to vary as in a straight line in accordance with the theory of a draft-tube, as suggested by F. Prasil (24). The dimensions of this bend are shown in Fig. 3. The differential equation required to represent the reduction in velocity, and the deceleration as a linear function of the distance measured along the axis of the bend is:

$$\tan \alpha = \frac{dr}{dx} = -\frac{r_0}{2x} \quad \text{.....................(19)}$$

in which r_0 = radius of the pipe; x = distance out from origin to the point where r_0 is measured; and, α = angle between the axis and the side-wall where r_0 is measured. Then, $2\,x\,dr = -\,r_0\,dx$; $2\,x\,dr_0 + r_0\,dx = 0$; $d\,(r^2_0\,x) = 0$; $r^2_0\,x = K =$ constant; $\pi\,r^2_0\,x = K' = A\,x$; and, $\frac{Q\,x}{V} = K'$. Therefore,

$$V = \left(\frac{Q}{K'}\right)x \quad \text{.........................(20)}$$

which represents a straight line.

If the velocity is differentiated with respect to time, the result will be deceleration; thus:

$$\frac{dV}{dt} = \left(\frac{Q}{K'}\right)\frac{dx}{dt} = \left(\frac{Q}{K'}\right) V \quad \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

This, again, is a linear equation in x, since $\frac{dx}{dt}$ is velocity. The velocity variation from the entrance to the mid-point of this bend is shown in Fig. 3 for a special case in which the rate of flow is 1.0 cu ft per sec.

TABLE 1.—SUMMARY OF PIPE-BEND DIMENSIONS

Bend No.	Vertex area, in square inches	Length of bend on axis, in inches	Generating angle of equivalent conical tube, in degrees	Radius of curvature of axis at vertex of bend, in inches
1..............	28.27	12.6	0	8
2..............	47.00	23.0	4° 20′	12
3..............	38.15	26.4	2° 20′	5
4..............	28.27	23.6	0	15
5..............	12.57	26.4	4° 20′	5

Table 1 shows a summary of the dimensions of Bends Nos. 1 to 5, each of which had an entrance diameter of 6 in., and (except Bend No. 1, in which, $L = 8$), a leg length of 15 in.

Construction of Pipe Bends.—All the pipe bends were constructed of transparent pyralin. After some preliminary experiments it was discovered that this material would become plastic when heated to a temperature of about 275° F, without losing its transparency, and when it was pressed into various shapes and cooled, it again became rigid.

The first stage in the construction of the bends was to make the moulds for pressing the material into the desired shapes. Full-sized drawings were made showing the intersections of planes passed through the bend parallel to, and at distances, in inches, of 1, 2, 3, etc., from the axis. The traces marking the intersection of the planes with the barrel of the pipe formed templates for marking the pieces of 1-in. sugar pine lumber so that they could be cut with a band-saw to the proper shapes. These pieces were then glued together and gouged out with a chisel to obtain the various cross-sections according to the different designs. Allowance was made in the construction of the moulds for the $\frac{1}{8}$-in. thickness of the pyralin between the two parts of each mould.

The material was heated in an electric oven at a temperature of about 275° F, and when the sheets became plastic they were placed between the moulds and pressed into shape. The pieces were put together with cement made by dissolving pyralin in acetone.

If the material was heated too much, it would lose its transparency; if not heated enough, it would wrinkle when pressed into warped surfaces; and, if heated too quickly, the surface became "frosty", and impaired the transparency. The "frosty" condition was overcome by rubbing the material with water sand-paper and polishing it with pumice.

Laboratory Experiments with Pipe Bends.—Fig. 4 shows the general arrangement of the apparatus during a test. The water, admitted at Point *K*, flowed through a conical expanding tube, and was deflected by a flat plate, into Tank *A*. A conical bottom, *C*, and guide-vanes in the bottom of the head-water tank directed the water, with or without a whirl component as desired, into a vertical pipe, *D*, 6 in. in diameter and 24 in. long, above the bend. The bend to be tested is indicated by *F*. After leaving the bend, the water flowed through a straight section of 6-in. pipe, 32 in. long, to a second tank, *B*. This tank was open at the top and fitted with a discharge valve at Point *H*.

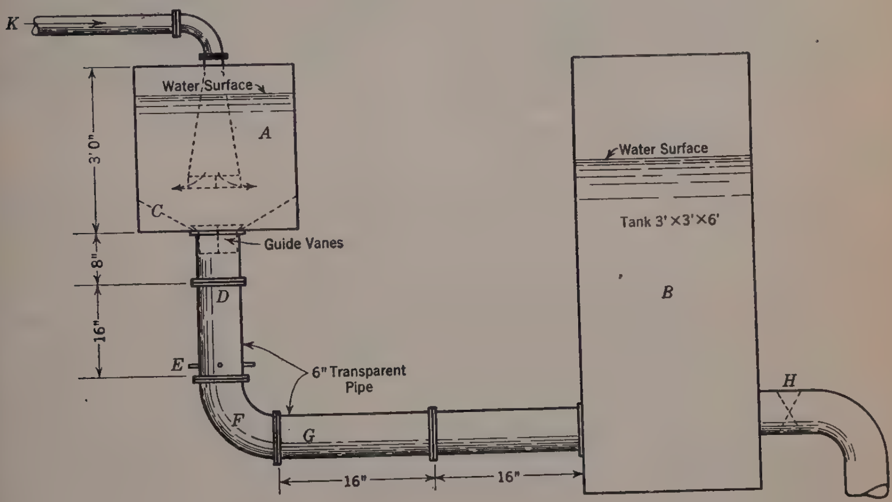


FIG. 4.—DESIGN OF LABORATORY APPARATUS.

The velocities of the water filaments were measured with a Pitot tube which was calibrated at the State University of Iowa, the Colorado Agricultural College, and at the Oregon State College. Its coefficient was found to be 0.86.

The rate of flow was determined by volumetric measurement. The water flowing from the test apparatus was collected in a concrete tank, and by use of a stop-watch and point-gage, readings were taken to determine the time required to raise the water surface to a given level in the tank so that the rate of flow could be established.

The bends were tested with two types of flow: (1) Direct flow without whirl component; and (2) helical flow, or that with a whirl component at the entrance to the bend. It has been stated that when the water approaches the bend with approximately uniform filamental velocities, there will be an induced double spiral as shown in Fig. 1. That this double spiral actually developed to a marked degree is indicated by Fig. 5(*a*), which is a view of the direct flow in Bend No. 3. At the vertex of the bend is shown a string stretched from the Pitot tube connections marked *B-B′*, and

attached to this string are several short pieces of yarn. The yarns show that the filaments of flow near the side-walls of the pipe, as at B and B', have velocity components toward the inside of the bend, whereas the filaments of

(*a*) DIRECT FLOW. (*b*) HELICAL FLOW.

FIG. 5.—DIRECT AND HELICAL FLOW IN BEND NO. 3.

flow near the center of the pipe have velocity components toward the outside wall of the bend. A double spiral flow has been induced, as shown in Fig. 1.

It is possible for a single spiral to exist in a pipe bend, with the water rotating in one direction or the other, depending on the flow at the entrance to the bend. This was shown in a series of experiments conducted by Messrs. Yarnell and Nagler (22) in 1934, in which the water at one side of the entrance to the bend had higher filamental velocities than on the other side. On the side where the axial velocities were small the water would move toward the inside, and on the side where the axial velocities were large the water would move toward the outside, of the bend, causing a single spiral instead of a double spiral.

Fig. 6 shows the velocity contours at different sections of Bends Nos. 1, 2, 3, and 4. In these diagrams the points marked A are at the outside of the bend, and the points marked A' are at the inside of the bend. In all cases the view is looking down stream. From these diagrams it is evident that, as the water approached the bend, the filaments of flow along the inside of the bend accelerated until they reached a maximum velocity about half-way around the bend. From that point the thread of maximum velocity moved to the outside of the bend.

Attention should be called to the unsymmetrical velocity contours in Bends Nos. 2 and 4. This was caused by having the cone in the head-water tank, A (Fig. 4), slightly off center. When this cone was centered the velocity contours were symmetrical about the center of the pipe, as shown for Bends Nos. 1 and 3 (see Fig. 6). It should be noted that any minor disturbance up stream from the bend was found to be reflected in the velocity distributions throughout the bend.

Curves were drawn for the combined velocity head and pressure head for all bends, as shown in Fig. 7. In the data the pressure has been recorded as p, for brevity, which was the indicated pressure head as given by the pressure

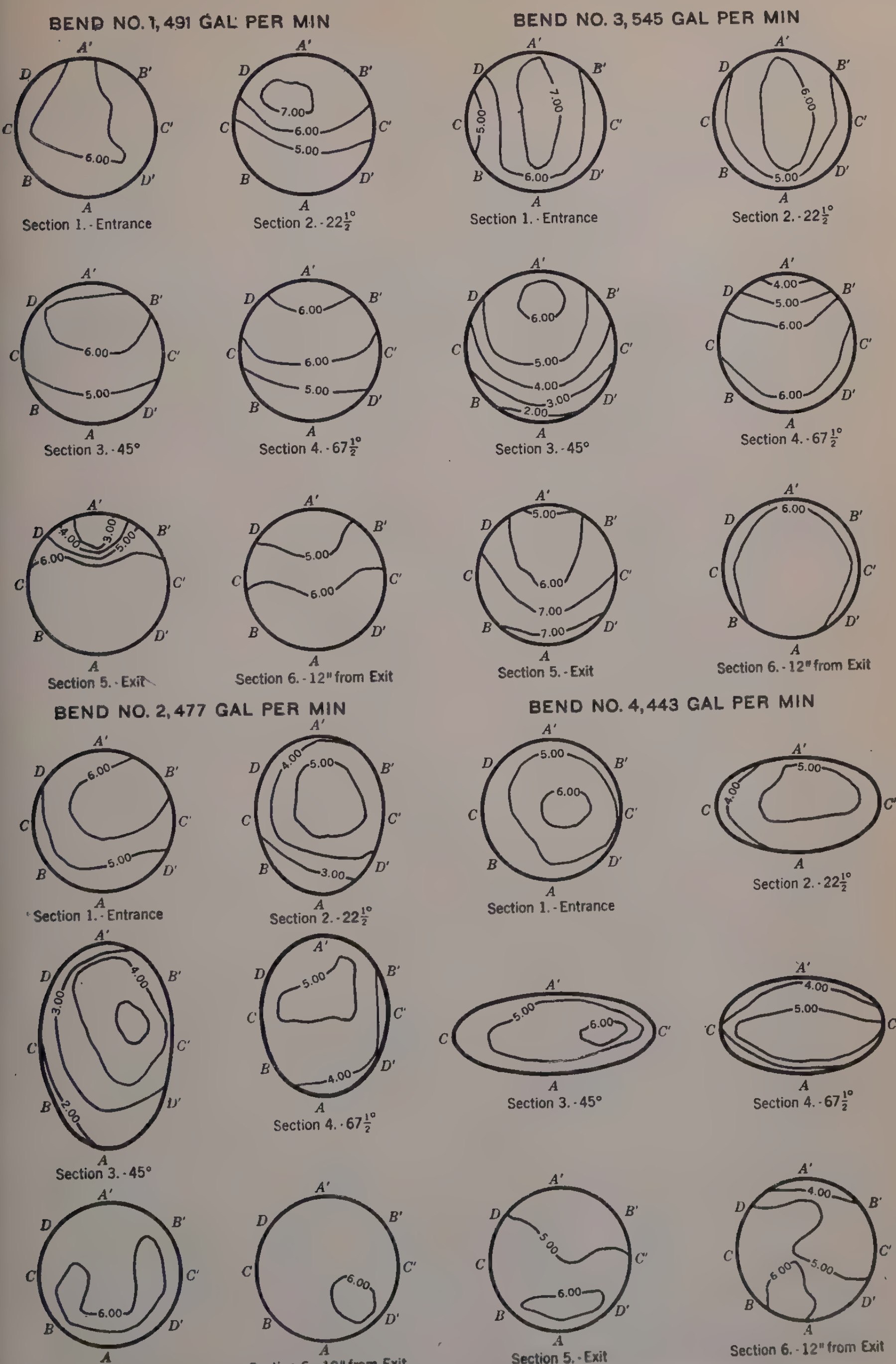


FIG. 6.—VELOCITY CONTOURS AT DIFFERENT SECTIONS OF EXPERIMENTAL BENDS.

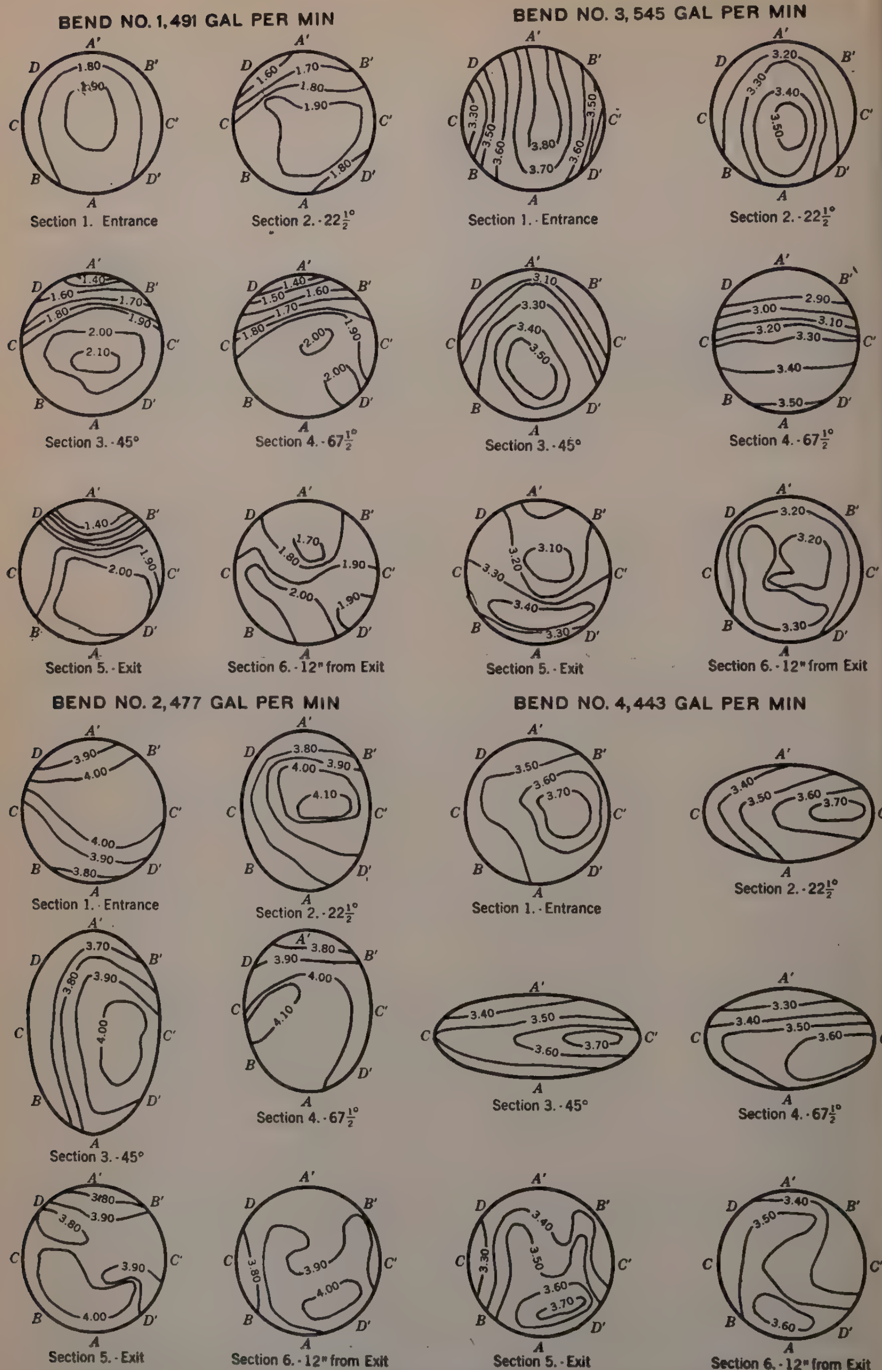


FIG. 7.—COMBINED VELOCITY-HEAD AND PRESSURE-HEAD CONTOURS AT DIFFERENT SECTIONS OF EXPERIMENTAL BENDS.

orifice of the Pitot tube. To obtain the corrected pressure head the equation for the Pitot tube was used:

$$V = 0.86 \ (2 g h)^{0.5} \qquad (22)$$

From Equation (22) h was computed for all the readings, added to $\frac{V^2}{2g}$, and plotted as shown in Fig. 7.

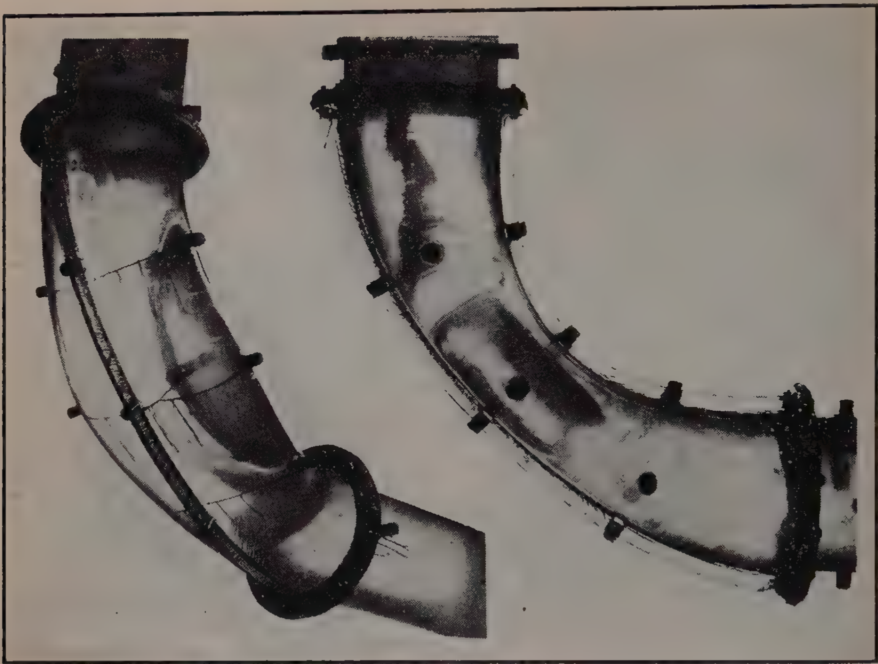
Tests were made to ascertain which of the bends induced the least loss of head for a given rate of flow. A bend was placed in the apparatus as shown at F in Fig. 4, and the water was admitted at a given rate so that the elevations of the water surfaces in Tanks A and B could be determined. This was repeated for all the bends, keeping the rate of flow constant. Bend No. 4 offered the least resistance to flow. As a check against this result, the bends were tested for head loss due to friction, by Messrs. Yarnell and Nagler (22) who found that, in a straight 6-in. pipe, the loss was $0.0302 \frac{V^2}{2g}$ per ft of pipe. The loss of head in the several bends, expressed in terms of the velocity head, and exclusive of frictional resistance, was as follows:

Bend No.	Loss of head in 90° bend (exclusive of friction)
1	$0.15 \frac{V^2}{2g}$
2	$0.15 \frac{V^2}{2g}$
3	$0.17 \frac{V^2}{2g}$
4	$0.13 \frac{V^2}{2g}$

This tabulation shows that Bend No. 4 offers less resistance to flow than any of the others, the advantage over Bend No. 3 being more than 30% of the velocity head. Its advantage over the standard short-radius bend was more than 15% of the velocity head. By an inspection of Figs. 5(*a*) and 8(*a*) it will be apparent that the induced spirals were more pronounced in Bend No. 3 than in Bend No. 4. Bend No. 4 has an additional advantage over the other bends when applied to elbow draft-tubes because it is flattened properly to avoid excessive excavation. This was a fortunate finding in the light of draft-tube construction, because of the desirability of widening the tube laterally to reduce excavation costs.

Helical flow, or that with a whirl component at the entrance to the bend, was produced by placing a stationary wheel in the apparatus at Point D (see Fig. 4). Only one type of whirl was used, where the rotation appeared to be counter-clockwise, facing down stream.

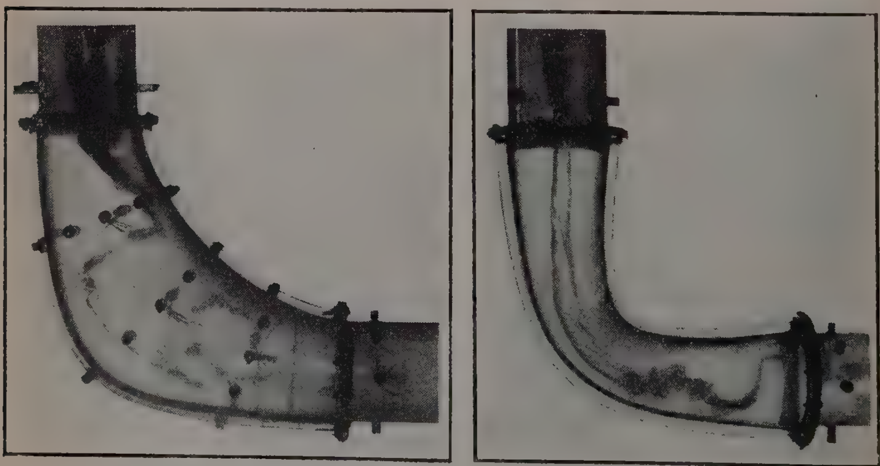
Bend No. 2 is shown in Fig. 9(*a*) with whirling component of flow at the entrance. The dark areas were made by admitting air in the stream of



(*a*) Direct Flow. (*b*) Helical Flow

Fig. 8.—Direct and Helical Flow in Bend No. 4.

inflowing water. The air bubbles moved into the region of least pressure, near the center of the whirl, and when a light was placed behind the pipe the air bubbles intercepted the light.



(*a*) Bend No. 2. (*b*) Bend No. 5.

Fig. 9.—Examples of Helical Flow.

Helical flow caused pulsations in all the bends, but it was at its worst in Bend No. 2. When the angle of whirl at the entrance to the bend was about 45° and the rate of flow was 0.8 cu ft per sec, the center of the vertex whipped back and forth from the inside to the outside of the bend at the rate of 240 times per min. This caused considerable vibration of the apparatus.

Fig. 5(*b*) shows helical flow in Bend No. 3. In this case the cross-sectional area increased from the entrance to the vertex of the bend, but not as much as in Bend No. 2. The principal difference in the two bends was that one had circular cross-sections and the other elliptical sections with a long axis in the plane of the bend. There was considerable vibration in Bend No. 2, whereas there was very little vibration in Bend No. 3 under similar conditions.

The center of the whirl in Bend No. 5, as shown in Fig. 9(*b*), was well defined in the upper (converging) half of the bend, but beyond the vertex of the bend the center of the whirl became disturbed, similar to that in the first half of Bend No. 3 (see Fig. 5(*b*)). Helical flow in Bend No. 4 (the bend which caused the least resistance to direct flow) caused some vibration, but not to the marked degree noted in tests of Bend No. 2.

The center of the vortex immediately ahead of the entrance to the bends had very little if any motion in the general direction of flow in the pipe. Just below the wheel at Point D (Fig. 4), there was a well-defined region near the center of the pipe in which the flow was toward the wheel. Mr. W. K. Ramsey (25) has expressed his belief that when there is whirling flow in a draft-tube, whether of straight or curved axis, there is a backward movement of water in the center of the whirl extending from the exit of the tube to the turbine runner. In none of the experimental bends or model draft-tubes described in this paper did the region of backward movement of water extend half-way around the bend, and in most cases it did not reach as far as the bend. Motion pictures were made of the flow through all the bends and draft-tubes to establish a permanent record of the conditions of flow.

No velocity measurements were made in any of the experimental bends or draft-tubes described in this paper when there was a whirl component of flow. When a Pitot tube is used for velocity measurements, the fluid should flow past the velocity and pressure openings in exactly the same manner as it did when the tube was calibrated, otherwise the velocity measurements may be of little value. The greater the angle of whirl in a pipe the greater will be the difficulty of obtaining correct velocity measurements with a Pitot tube; and particularly is this true if it is not possible to observe the direction of flow past the end of the tube.

EXPERIMENTS WITH MODEL DRAFT-TUBES

Design and Construction of Model Draft-Tubes.—During 1933 the Federal Government appropriated funds for the construction of a dam at Bonneville, Ore., across the Columbia River. The model of the draft-tubes for this dam were made of transparent pyralin, the same as that of the pipe bends which have been described. Moulds were made of wood, and these moulds

were used to press the sheets of transparent material into the desired shapes. Some trouble was encountered in pressing the flat sheets into such warped shapes, but for the most part a fairly workmanlike and finished model draft-tube was obtained. Fig. 10 shows the moulds for a typical draft-tube bend.

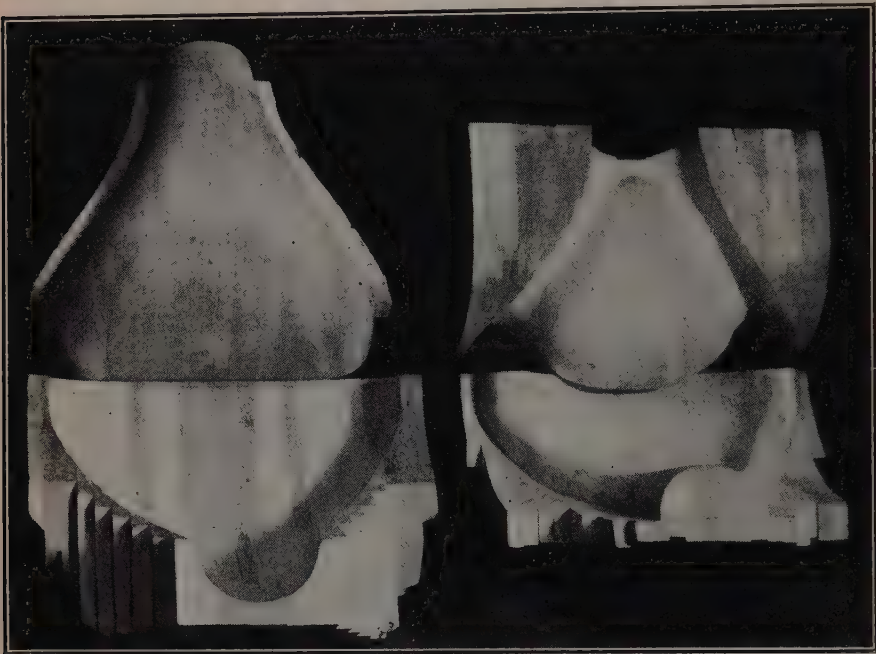


FIG. 10.—MOULDS FOR BEND OF MODEL DRAFT-TUBE NO. 1.

The mould on the left was for the lower half of the bend of Model Draft-Tube No. 1, and that on the right was for the inner half of the bend of the same tube.

Pyralin is much superior to glass as a transparent material of which to build model draft-tubes because of its durability and ease of fabrication. Prasil (26) used glass in the construction of a draft-tube with a straight center line, but no mention was made of the use of Pitot tube connections in his experiments. Piezometer connections can be made much more easily of pyralin than of glass, and with less trouble due to breaking.

The design of the model draft-tubes for the U. S. Engineer Department was made in the office at Portland, Ore. No information as to their design was available to the writer other than the detail drawings, such as shown in Fig. 11, Draft-Tube No. 1, and the cross-sectional areas at different points along the axis of the tubes, as shown in Table 2. The cross-sections of Tube No. 1 are not circular at any point along its axis except at the entrance. The tube expands at the same rate as a cone having a 6° generating angle. There is no "splitter", and the bearing partition does not enter the

bend. The distinctive feature of this bend is the manner in which the walls are "dished" down on the inside of the bend and lower leg.

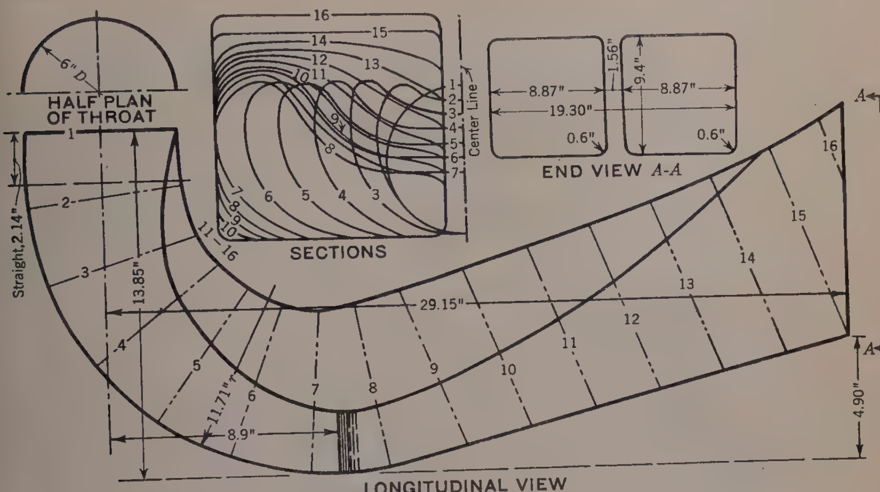


FIG. 11.—DETAILS OF MODEL DRAFT-TUBE NO. 1.

Tube No. 2 has a 6° cone for the vertical leg. The lower leg is rectangular in section. The "splitter" extends completely around the bend, as shown in Fig. 12, and the bearing partition terminates at about the middle of the bend. This tube is similar to the draft-tubes built at Ryburg-Schwörstadt (27) on the Rhine, in Germany.

TABLE 2.—AREAS OF DIFFERENT SECTIONS OF MODEL DRAFT-TUBES

Section No.	Area of 6° cone, in square inches	Areas of Sections, in Square Inches, for Different Model Draft-Tubes			Section No.	Area of 6° cone, in square inches	Areas of Sections, in Square Inches, for Different Model Draft-Tubes		
		Tube No. 1	Tube No. 2	Tube No. 3			Tube No. 1	Tube No. 2	Tube No. 3
1	28.3	28.3	28.3	28.3	9	84.5	76.0	83.7	68.5
2	33.6	33.6	33.6	33.6	10	91.7	88.7	96.5	81.0
3	39.4	39.0	39.4	39.4	11	103.0	100.3	104.5	94.5
4	45.8	46.8	45.6	45.8	12	113.0	111.0	114.0	107.5
5	52.6	53.5	54.3	47.0	13	124.0	122.3	124.0	120.5
6	60.0	61.5	60.0	45.5	14	135.0	132.8	135.5	134.0
7	67.7	60.0	66.4	45.5	15	146.0	143.0	148.5	146.5
8	75.7	65.0	75.0	55.5	16	159.0	153.0	.....	.....

Tube No. 3 is dished on the inside of the bend, but to a less degree than Tube No. 1. The vertical leg widens in a direction normal to the plane of the bend, and there is no "splitter." The areas of the sections at different points along the axis of the tube are about the same as that for a 6° cone as given in Table 2. The bearing partition extends almost completely around the bend, as shown in Fig. 13.

Design of Model Draft-Tube No. 4.—This tube was designed for the same setting as the model draft-tubes for the U. S. Engineer Department. The given data were: (1) Elevation of entrance of draft-tube; (2) entrance

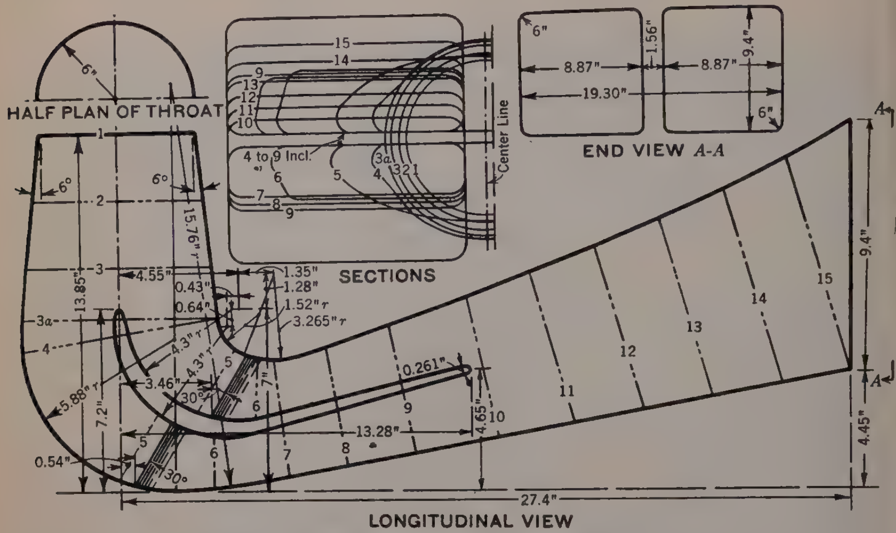


FIG. 12.—DETAILS OF MODEL DRAFT-TUBE NO. 2.

diameter; (3) elevation of top edge of outlet; (4) maximum width; and, (5) normal discharge. The vertical leg of the tube was made as long as it could be conveniently, so that the velocity would be reduced before the water

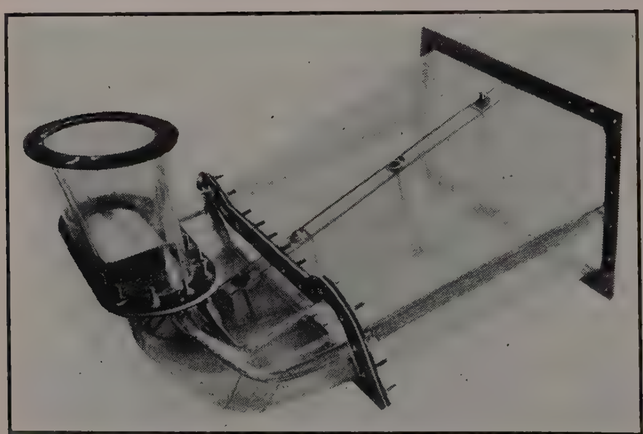


FIG. 13.—MODEL DRAFT-TUBE NO. 3

entered the bent part of the tube. It was trumpet-shaped, as suggested by Prasil (24). This type of vertical leg appeared to have certain advantages since the deceleration at the entrance of the tube was small and there seemed

to be less likelihood of inducing cavitation. The length of the vertical leg used was the same as that of Tube No. 2—7.23 in.—and the entrance was 6 in. in diameter.

The angle of flare of 7° was selected after considering earlier tests by the writer and by Hofmann (8). Draft-Tube No. 2, which showed a very high efficiency, had an angle of flare of 6 degrees. Hofmann reported that an angle of flare of 8.5° could be used to advantage, and common practice in draft-tube design has shown that a flare somewhat greater than 6° could be used.

For purposes of design, the rate of flow was taken as 1.0 cu ft per sec. The mean velocity in the 6-in. entrance to the tube was 5.08 ft per sec. Then, at Section 7.23, the radius of the tube became: $r_o = 3.00 + 7.23 \tan 7° = 3.89$ in.; and, $V = \dfrac{Q}{A} = \dfrac{144}{\pi\,(3.89)^2} = 3.03$ ft per sec. These values are to be substituted in Equation (20) which was derived by use of the theory suggested by Prasil (24). At the entrance to the tube (see Fig. 14), x was found to be 17.94 in. and, consequently, the rate of deceleration at that point becomes $\delta_o = \left(\dfrac{Q}{K'}\right)^2 x_o = [(5.08)\ (12) \div 17.94]^2\ (17.94 \div 12) = 17.25$ ft per sec^2; and at Section 7.23, $\delta_{7.23} = (3.4)^2 \times \dfrac{(17.94 - 7.23)}{12} = 10.3$ ft per sec.^{2} The reduction of velocity per inch of distance along the axis is, $\dfrac{(5.08 - 3.03)}{7.23} = 0.2835$ ft per sec per ft; the velocity at Section 10 is, $V_{10} = 5.08 - 10\,(0.2835) = 2.25$ ft per sec; and, $\delta_{10} = 17.25 - \dfrac{10}{7.23}\,(17.25 - 10.3) = 7.63$ ft per sec.^{2}

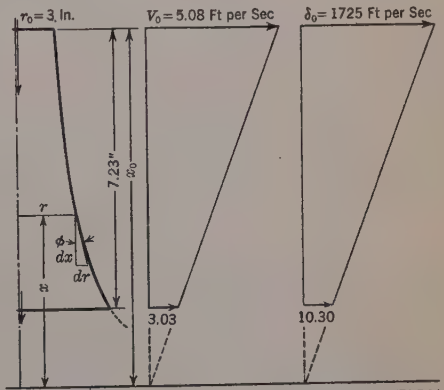


FIG. 14.—STRAIGHT-LINE VELOCITY AND DECELERATION CURVES IN PRASIL DRAFT-TUBE.

From Section 10 the increase in area of the tube is the same as that for a 7° cone. It should be noted that where the change occurs from a Prasil section to a conical section, there is an abrupt change in the deceleration, as shown in Fig. 15. The deceleration in a cone is given by the equation:

$$\delta_x = \frac{2 \tan \phi\ (V_x)^{2.5}}{r_o\ V_o^{0.5}} \qquad (23)$$

in which, r_o = radius of cone at entrance of tube; V_o = entrance velocity of water; and, V_x = velocity of water at any other section where deceleration is

desired. The radius of a circle having an area of 0.448 sq ft at Section 10, is 4.53 in., which, substituted in Equation (23), yields, $\delta_{10} = \dfrac{2\,(\tan 7^\circ)\,(2.25)^2}{\dfrac{4.53}{12}}$ $= 3.3$ ft per sec². Thus, the deceleration drops suddenly from 7.6 ft per sec.² to 3.3 ft per sec.², as shown in Fig. 15. The deceleration at the entrance of the Prasil tube was computed to be 12.16 ft per sec.², as compared to 15.3 ft per sec.² at the entrance of a 6° cone.

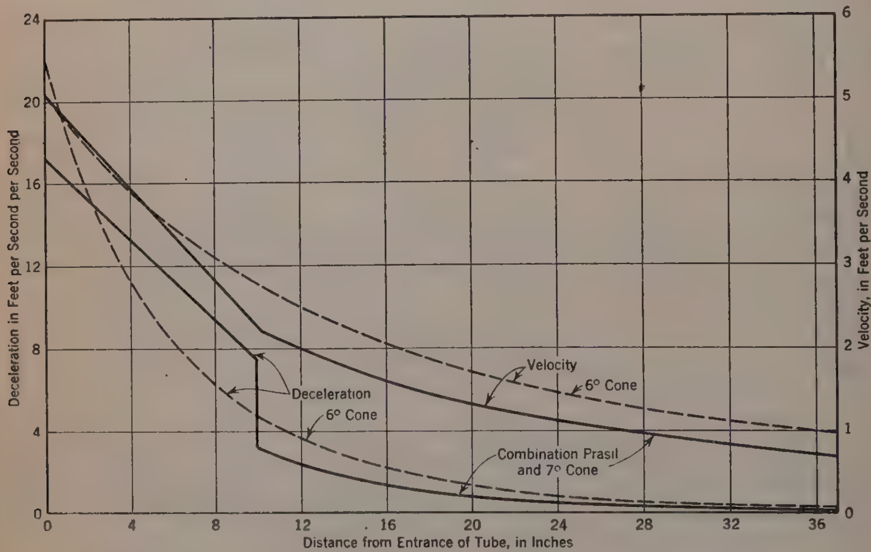


FIG. 15.—VELOCITY AND DECELERATION IN MODEL DRAFT-TUBE NO. 4

The horizontal leg of the draft-tube, as designed, was rectangular in section. The upper wall was a plane surface, the sides were parallel, and the floor slope was varied in the direction of flow to provide for the increase in area necessary to obtain the flare equivalent to a 7° cone. Fig. 16 shows the details of the model draft-tube.

At the upper end of the bent part the cross-sections were circular and at the lower end they were rectangular. The side curves were drawn in to obtain a smooth curve as shown in the plan view. The outside curves were drawn as smooth lines without any mathematical relation from which to establish them.

The cross-sections of the bend in the draft-tube were patterned after the energy and velocity contours obtained for the test on Bend No. 4 and shown in Sections 4, Figs. 6 and 7. The general plan of the design of the cross-sections of the bend was as follows: At the beginning of the bend, use a circular cross-section; at 22½°, use an elliptical section; at 45°, use an ellipse of greater eccentricity for the inside than for the outside part of the section; and, at the end of the bend, use a rectangular section.

An equation suitable for obtaining such curves as could be used for the sections between the vertex and the end of the bend where the transition must be made from an ellipse to a rectangle, has the general form:

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

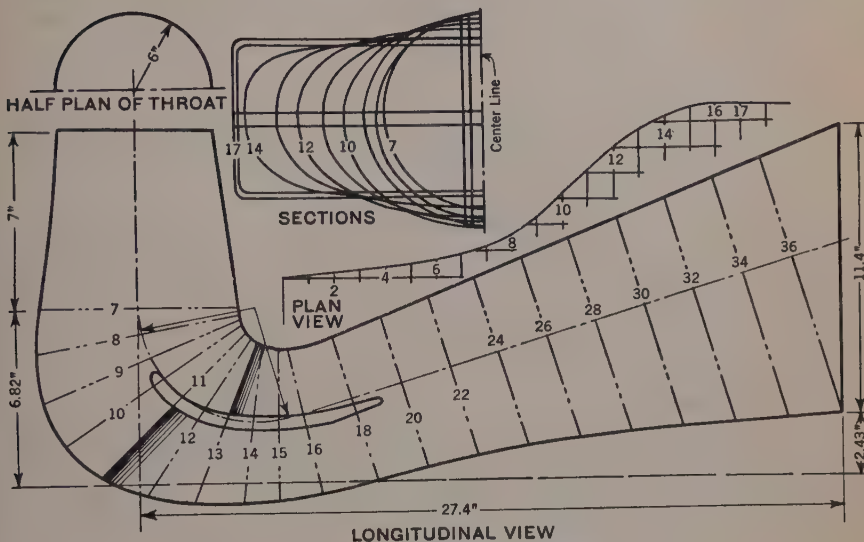


FIG. 16.—DETAILS OF MODEL DRAFT-TUBE NO. 4.

in which $0 \leq x \leq a$; $0 \leq y \leq b$; and the curve is limited to the first quadrant. As the values of a and b could be scaled from Fig. 16, and the values of the exponent, n, could be varied to obtain the approximate total cross-section that would give the required area, an exact definition of the cross- sections could be given. However, this refinement was not necessary, and no further mathematical analyses of the sections of the bends were made.

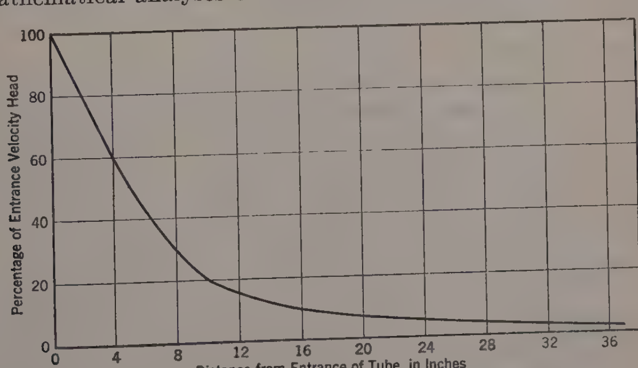


FIG. 17.—PERCENTAGE OF VELOCITY HEAD REMAINING AT VARIOUS SECTIONS OF MODEL DRAFT-TUBE NO. 4.

Fig. 17 shows the percentage of the entrance velocity head that remains in the water at different sections of Model Draft-Tube No. 4, the flow at the

entrance being 1.0 cu ft per sec. From this curve it is evident that the major portion of the velocity head should be reclaimed in the vertical leg of the tube.

Laboratory Tests on Model Draft-Tubes.—The tests of the model draft-tubes were made with the same apparatus that was used in testing the experimental pipe bends (see Fig. 4). The only change in this apparatus was to fasten the lower, or outlet, end of the draft-tube to Tank *B*. The throat of the draft-tube was fastened to the same flange to which the entrance of the pipe bend was fastened, as at Point *E*.

The water that flowed through the draft-tube was measured by volume to establish the rate of flow, which was regulated by the valves in the inlet pipe, *K*, and in the outlet pipe, *H*.

Each draft-tube was tested first with a direct flow of the entering water. The pressure at the throat of the tube was obtained through four piezometer connections spaced at equal intervals around it, and the pressure at the exit of the tube was obtained from the elevation of the water surface in the tank, *B*. Pitot tube measurements of velocities in the lower leg of two of the draft-tubes were recorded for several different rates of flow, both with and without the "splitter" in place.

To obtain whirling flow in a draft-tube, a special set of curved vanes was used above the entrance to the tube replacing the guide-vanes shown in Fig. 4. Air was admitted with the water to assist in observing and photographing the position of the center of the whirl. Motion pictures were taken of all the different conditions of flow[8a].

The efficiency of a draft-tube as a regainer of static pressure has been defined in various ways. To the writer it seems logical to define it as the ratio of the head actually recovered to the head available for recovery, or,

$$e = \frac{\text{Regain}}{\text{Throat velocity head}} = \frac{\frac{p_2}{w} - \frac{p_0}{w}}{\frac{V_0^2}{2g}} \quad \ldots\ldots\ldots\ldots\ldots(25)$$

in which p_2 = the pressure head at the exit of the tube; p_0 = the pressure head at the throat of the tube; and V_0 = the velocity at the throat of the tube. The Power Test Code of the American Society of Mechanical Engineers (28) recommends deducting the velocity head at the exit of the tube in computing efficiency. Luksch (6) stated that if the discharge velocity is deducted in computing the efficiency of a draft-tube the result would be indeterminate if applied to a straight cylinder, and he concluded "that it is not justified to account for the discharge energy out of the draft tube, $\frac{V^2}{2g}$, as acting in the sense of an improvement of the efficiency of a draft tube."

No method was devised by the writer for obtaining the efficiency of draft-tubes for conditions of whirling flow. When the water had a whirling motion

[8a] Any one interested may borrow these motion pictures by addressing a request to the Secretary of the Society.

at the entrance to the tube, the pressure as indicated by the piezometers at the walls of the pipe gave no indication of the mean pressure at that point. Under the conditions of testing as shown in Fig. 4, the pressure at the center of the entrance pipe of the draft-tube was below atmospheric, whereas the pressure near the walls of the entrance pipe was decidedly above atmospheric.

Table 3 shows the efficiencies of the several draft-tubes under conditions of direct flow. Draft-Tube No. 2 showed the highest efficiency (about 65%), indicating that it actually converted into pressure head about 65% of the velocity head at the throat of the tube.

TABLE 3.—SUMMARY OF OBSERVED EFFICIENCIES OF MODEL DRAFT-TUBES

Flow, Q, in cubic feet per second (1)	Throat velocity, V, in feet per second (2)	Velocity head, $\frac{V^2}{2g}$ (3)	PRESSURES: Entrance (4)	PRESSURES: Exit (5)	Head regained by tube (6)	Percentage, efficiency (7)
(*a*) TUBE NO. 1, COMPLETE						
1.66	8.74	1.185	3.360	3.910	0.550	46.5
1.66	8.74	1.185	3.310	3.870	0.560	47.2
1.65	8.68	1.170	3.310	3.870	0.560	47.8
1.31	6.90	0.740	3.800	4.150	0.350	47.3
Average	.....	.....	.....	.....	.....	47.2
(*b*) TUBE NO. 2, COMPLETE IN PLACE						
0.328	1.72	0.046	3.020	3.050	0.030	65.0
0.725	3.81	0.226	4.070	4.212	0.142	62.8
1.684	8.86	1.220	2.880	3.690	0.810	66.3
1.720	9.05	1.270	2.910	3.740	0.830	65.4
1.250	6.60	0.675	3.424	3.864	0.440	65.1
Average	.....	.....	.....	.....	.....	64.9
(*c*) TUBE NO. 2, WITH HORIZONTAL SPLITTER REMOVED						
1.270	6.65	0.689	3.502	3.937	0.435	....
Average	.....	.....	.....	.....	.....	62.9
(*d*) TUBE NO. 3, COMPLETE IN PLACE						
0.844	4.44	0.381	3.017	3.160	0.153	46.3
0.975	5.13	0.409	2.785	2.980	0.195	47.8
0.995	5.24	0.426	2.785	2.980	0.195	45.7
1.160	6.10	0.578	3.195	3.470	0.275	47.7
1.255	6.62	0.680	3.596	3.920	0.324	47.7
Average	.....	.....	.....	.....	.....	47.0
(*e*) TUBE NO. 4 COMPLETE IN PLACE						
0.650	3.41	0.181	3.663	3.775	0.112	62.0
0.817	4.30	0.288	3.642	3.822	0.180	62.5
0.822	4.32	0.291	3.674	3.855	0.181	62.2
0.828	4.36	0.295	3.640	3.826	0.186	63.0
0.830	4.37	0.297	3.661	3.846	0.185	62.3
1.000	5.25	0.427	3.648	3.922	0.274	64.0
Average	.....	.....	.....	.....	.....	62.7

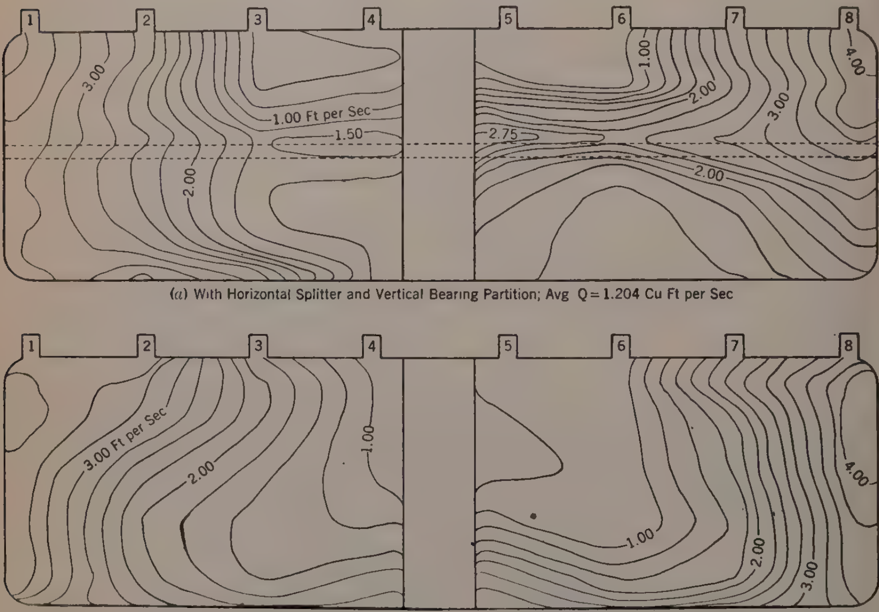
When the horizontal "splitter" in Draft-Tube No. 2 was removed, the tests, summarized in Table 3, showed that the efficiency was lowered about 2 per cent. Of course, some of the cross-sectional areas of the bend were increased about 10% or 15%, but the writer does not believe this could account entirely for the reduced efficiency.

Draft-Tubes Nos. 1 and 3 had an efficiency of about 47 per cent. These two tubes had the central portion of the top surface of the entrance to the lower leg "dished down" very much in the same manner as shown in drawings of a draft-tube developed by G. A. Jessop, Assoc. M. Am. Soc. C. E. (29). Although the lowering of the roof part of the entrance of the lower leg might have been expected to eliminate the dead-water space common in some elbow draft-tubes and thus show good efficiency as a regainer the results were not very good.

There appeared to be no marked difference in the efficiency of the tubes for the different rates of flow. The efficiency was about the same for the low-

est rate of flow as for the highest rate; therefore, the fluctuations in efficiency as shown in Table 3 were attributed to experimental error. In tests on draft-tubes having throat diameters of 3 in., Gibson and Labrow (30) concluded that the efficiency was sensibly independent of the velocity for velocities exceeding about 5 ft per sec. In an investigation of the transformation of water velocity into pressure with diffusers having a throat diameter of 75 mm, K. Andres (31) found that the efficiency exhibited by any tube was independent of the velocity over a range of velocities between 30 and 130 ft per sec.

The Pitot tube measurements of the velocities near the entrance (Section 10) of the lower leg of Draft-Tube No. 2, both with and without the "splitter", and facing in the direction of flow, are shown in Fig. 18. From these curves,



(*a*) With Horizontal Splitter and Vertical Bearing Partition; Avg Q = 1.204 Cu Ft per Sec

(*b*) With Vertical Bearing Partition; Avg Q = 1.193 Cu Ft per Sec

FIG. 18.—VELOCITY CONTOURS IN LOWER LEG OF MODEL DRAFT-TUBE NO. 2.

it is evident that the "splitter" had a material effect on the filamental velocities, but did not eliminate entirely the region of lessened velocities near the inner part of the exit of the bend. From the curves it appears that the velocities were higher near the right side of the tube, looking down stream, than on the left side. This was found to have been caused by the flaring tube in Tank *A* (Fig. 4), being about $\frac{1}{2}$ in. off center. When the flaring tube was adjusted to the center of Tank *A*, its two sides showed approximately symmetrical conditions of flow.

Fig. 19 shows velocity contours at a section (Section 20) near the exit of the bend of Draft-Tube No. 4, facing in the direction of flow. It is evident that there was considerable dead water near the inside of the bend, a condition that did not exist in Tube No. 2. The reason for the occurrence

of the dead water was attributed to too great a distance between the top and bottom walls of the exit of the bend, a measurement hereafter designated as the Δ-distance of the tube.

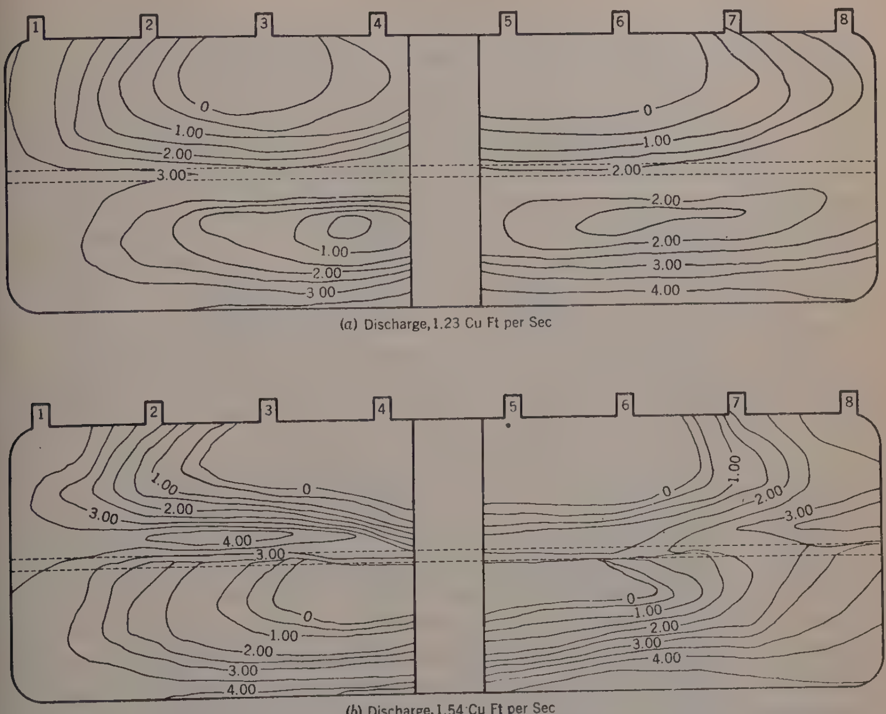


FIG. 19.—VELOCITY CONTOURS IN LOWER LEG OF MODEL DRAFT-TUBE NO. 4.

In Draft-Tube No. 4 the "splitter" was designed to extend only part way around the bend. An extension was made so that it would extend completely around the bend, but this made no difference in the efficiency of the tube. Thus, the conclusion was drawn that the "splitter" need not extend completely around the bend to yield good efficiency. It would be better with whirling flow if the "splitter" were limited to a position about as shown in Fig. 16.

With whirling flow there was considerable turbulence throughout the length of all the draft-tubes. This was at its worst with those tubes having the "splitter", such as Tube No. 2, if the "splitter" extended completely around the bend. An attempt was made to correct the turbulence in Tube No. 2 by placing a "twisted fin" on the upper end of the "splitter." The results of this experiment are shown in Table 4.

The loss of head in flow through the tube was somewhat reduced by the "fin", but the difference was not great. It is known that the whirl component of the filamental velocities in draft-tubes may be clockwise at one time and counter-clockwise at another, depending upon the changes in head and gate-

opening It is possible that the whirl may be clockwise near the walls of the draft-tube and counter-clockwise near the center of the tube. It is evident, therefore, that the use of any such deflector as described herein for correction of whirl in a draft-tube is not practicable.

TABLE 4.—ELEVATIONS OF WATER SURFACES IN TANKS *A* AND *B* FOR DRAFT-TUBE NO. 2, WITH AND WITHOUT SPECIAL FIN FOR CORRECTING WHIRL

WITH FIN IN PLACE (RATE OF FLOW, 0.961 CUBIC FOOT PER SECOND. MEAN VELOCITY AT ENTRANCE OF TUBE, 5.11 FEET PER SECOND)			WITHOUT FIN IN PLACE (RATE OF FLOW, 0.975 CUBIC FOOT PER SECOND. MEAN VELOCITY AT ENTRANCE OF TUBE, 5.13 FEET PER SECOND)		
Water-Surface Elevations		Lost head	Water-Surface Elevations		Lost head
Tank *A*	Tank *B*		Tank *A*	Tank *B*	
4.327	2.034	2.293	4.410	2.020	2.390
4.328	2.035	2.293	4.418	2.027	2.391
4.335	2.037	2.298	4.408	2.012	2.396
4.341	2.024	2.317	4.407	2.027	2.380
4.335	2.035	2.300	4.410	2.012	2.398
4.339	2.044	2.295	4.420	2.025	2.395
4.310	2.015	2.295	4.415	2.024	2.391
4.345	2.025	2.320	.....	.....	.....
Average.....	.....	2.310	Average.....	.....	2.391

RECOMMENDATIONS FOR THE DESIGN OF ELBOW DRAFT-TUBES

The Vertical Leg.—An elbow draft-tube should have a trumpet-shaped vertical leg as long as the suction head and the construction costs will permit. A long vertical leg seems to be justified by the fact that the bent part of an elbow draft-tube induces spiral flow and, consequently, friction and eddy losses which vary approximately as the square of the velocity of the flowing water. If there is a vertical leg in a draft-tube, the induced spiral velocities in the bend and the resultant hydraulic losses should be smaller than in an elbow draft-tube which has no vertical leg.

As indicated in Fig. 17, there is a greater conversion of kinetic energy to potential energy per foot of draft-tube where the mean velocity is high than where it is low. This is due to the fact that the velocity head varies as the square of the velocity. In the case of Model Draft-Tube No. 4, it would be theoretically possible, under ideal conditions, to obtain an efficiency of more than 80% with the tube cut off at Section 10. Of course, it would be impossible to obtain ideal conditions where there would be no friction and eddy losses, but there can be a closer approach to ideal conditions in a straight pipe than in a curved pipe. Friction on pipe walls cannot be eliminated, and centrifugal force is inherent to change in the direction of flow. These two in combination induce spiral flow as shown in Fig. 5(*a*). The induced spirals cause hydraulic losses that would not occur in a straight pipe.

There is a possibility that the induced spirals due to the bend in an elbow draft-tube may approximately offset the undesirable whirl components of flow from a turbine runner. It is very doubtful, however, whether a designer, either by chance or by careful planning, could discover the combination of conditions needed for this delicate balance. Further research along

this line may be expected in the future, since rational and experimental evidences have been submitted (32) (33) (34) by many able engineers in support of their conclusions that the form of draft-tube affects the performance of the runner.

The trumpet shape of the vertical leg, such as that suggested by Prasil (24) seems to be preferable to the straight circular cone. The experiments of Hofmann (8) Gibson and Labrow (30) and Andres (31) indicate that the trumpet-shaped tube has as great an efficiency as a straight cone, or greater. The cone-shaped tube has a further disadvantage that at the entrance to the tube the deceleration is comparatively high and is changing rapidly. This occurs in a region where cavitation troubles may arise. Research at the Massachusetts Institute of Technology (35) shows that cavitation is a mechanical phenomenon in which there is a rapid formation and collapse of small cavities in the flowing water. It appears that the rapidly changing decelerating force at the entrance of a conical tube would make a turbine runner more susceptible to cavitation than the lesser decelerating forces in the trumpet-shaped draft-tubes. It seems, therefore, that the vertical leg of a draft-tube should be trumpet shaped.

The Bend.—In the test of the experimental pipe bends it was shown that Bend No. 4, which was widened in the direction normal to the plane of the bend, offered the least resistance to flow. This was a fortunate finding, because of the ease with which this type of bend can be adapted to an elbow draft-tube.

In the report of the Bureau of Standards (16) on the investigation of literature on draft-tubes, the following statement was made:

"* * *, some conditions for improving the efficiency of elbow type draft tubes will be stated, which probably are correct, since apparently they are all met with in practically all the elbow tubes of the better design. First, the curvature of the inside wall should be sharp, reducing the length of the curve. Secondly, the depth of the tube at the section where the water is deviated should be small. Third, where the inside wall has considerable curvature, the areas of the sections from point to point should be nearly constant."

It is interesting to note that Bend No. 4 essentially fulfills all these conditions, since the radius of curvature of the inside wall is smaller than that of the outside wall, all the cross-sectional areas are alike, and the pipe is flattened in the plane of the bend.

This flattening, arranged so that the depth of the draft-tube at the exit of the bend is small, seems to be of considerable importance. Draft-Tube No. 2 (Fig. 12) has less depth at the exit of the bend than Draft-Tube No. 4 (Fig. 16). Otherwise, the shapes are quite similar Draft-Tube No. 2 had no dead-water space in the lower leg (Fig. 18), whereas Tube No. 4 had considerable dead-water (Fig. 19), with a resultant lowered efficiency.

Luksch (6) has shown that with direct flow in a trumpet-shaped draft-tube, the maximum efficiency occurred when the impact plate was placed so that $\frac{\Delta}{D_0} = 0.27$, in which Δ = the distance between the impact plate and the bottom of the draft-tube; and D_0 = the throat diameter of the draft-tube.

With values of $\frac{\Delta}{D_0}$ greater than 0.27, the water would jump free from the inner face of the tube and permit the formation of a dead-water zone. This same phenomenon probably occurs in an elbow draft-tube, because the bottom part of the bend acts in a manner similar to an impact plate.

It was stated that the radius of curvature of the inside wall of the bend should be smaller than that of the outside wall. Just what relation these radii should have to the throat diameter of the tube is not known, but some notion may be had from the experiments of Hofmann (8) who showed that if an impact plate is used below a conical draft-tube with rounded bottom edges, the most favorable ratio for rounding is, $\frac{r}{D_0} = 0.85$, in which D_0 = the throat diameter of the tube; and r = the radius of the rounded bottom edges of the draft-tube. He showed, furthermore, that $\frac{r}{D_0}$ may vary between 0.67 and 1.00 without much change in efficiency of the tube. This indicates that the elbow draft-tube may have a wide range of values of the radius of curvature for the inner walls of the bend without losing materially in efficiency.

The shape which should be given the curve shown as the plan view, in Fig. 16, seems to be of no great importance, except that preferably, it should be a smooth curve. In all the model draft-tubes tested by the writer the water had comparatively high velocities of flow along the sides of the tubes, as shown in Figs. 18 and 19.

The cross-section at the entrance of the bend should be circular, and, at the exit, it could well approach that of a rectangle. The intermediate sections should be flatter on the inside of the bend than on the outside.

A "splitter" should be used in the bend of an elbow draft-tube because of its tendency to assist in getting the water around the bend in an orderly manner. Tests on Draft-Tube No. 2 gave about 2% higher efficiency with the "splitter" than without it. In the Lilla Edet (17) tests, where the draft-tube had practically no vertical leg, the "splitter" increased the efficiency slightly.

It is undesirable to extend the "splitter" completely around the bend. Tests with Draft-Tube No. 4 were made first with the "splitter" beginning at about one-third the way around the bend (Fig. 16) and then extended so that it began at the very entrance to the bend. No noticeable change in the efficiency was observed. Caflish (36) tested a turbine of the Francis type for which he showed that the "splitter" reduced the efficiency when extended into the vertical leg of the draft-tube. This reduction was to be expected, as the turbine caused a whirl component of flow at the part gate-opening. At the larger gate-openings the "splitter" seemed to have little effect on the efficiency. Had it been extended only part way around the bend, such as at the Kembs Power Plant (37), on the Rhine, there probably would have been less turbulence at part-gate operation.

Professor Spannhake (38) has mentioned the desirability of whirl in the draft-tube in the positive sense of the rotation of the runner, this whirl allowing the runner blades to be designed for lower relative velocities and, in consequence, for reduced possibility of cavitation. If this should be the practice, then the extension of the "splitter" into the vertical leg is undesirable. However, it does seem desirable to have both the "splitter" and the vertical bearing partition, commonly built in the lower leg of the tube, extend some distance into the bend.

The Lower Leg.—The length, and particularly the width, of the lower leg of an elbow draft-tube greatly influence the design of the power house. Generally, the width of the draft-tube controls the spacing of the turbines, and the type and size of the plant determine whether or not a bearing partition shall be used in the lower leg of the draft-tube as a means of giving stability to the power-house structure.

The proportions that should be given the lower leg of elbow draft-tubes have been the subject of an investigation (39) for the Wheeler Dam, in the Tennessee Valley. Several tubes were tested with the result that the best efficiency in each tube was obtained when the length of the lower leg was the longest tested. In Plate A-6 of the report (15) of the National Electric Light Association on draft-tubes, are given the results of tests on changing the length of the lower leg of an elbow draft-tube by the Newport News Shipbuilding and Drydock Company. Leg lengths of 2.50, 3.00, and 3.71 times the throat diameter of the tube were used, and the efficiency of the tube was increased with each increase in leg length. This does not mean that the length could be increased indefinitely with a continued increase in efficiency. Hofmann's tests (8) on conical tubes led him to conclude that at first an increase of the length of a draft-tube will cause an improvement in efficiency and, finally, that the wall friction becomes too great and causes a decrease in efficiency. Further experiments are required to determine what the optimum length of the lower leg of an elbow draft-tube should be.

Summary and Conclusions

Experiments with Pipe Bends.—

(1) The loss of head due to the flow of water through a standard 6-in. pipe bend of 8-in. axial radius was 0.15 $\frac{V^2}{2g}$ as compared to values of 0.13 $\frac{V}{2g}$ to 0.17 $\frac{V^2}{2g}$ for the other pipe bends tested. The specimen that offered the least resistance to the flow of water was flattened in the plane of the bend. Its superior performance probably was due to the fact that fewer induced spirals occurred in this form than in the other bends.

(2) The specimen that was widened in the plane of the bend had more vibration than the other bends when tested under conditions where the entering water had a whirl component of velocity.

Experiments with Model Draft-Tubes.—

(3) The efficiency of the model draft-tubes of 6-in. throat diameter as a device for converting velocity head into pressure head varied between 47% and 65% when the efficiency was defined as the ratio of the head actually recovered to the head available for recovery. There was no appreciable change in the efficiency of the model draft-tubes for changes in rate of flow.

(4) When there was considerable whirl component of the velocities of flow in the draft-tubes, there was a central rotating core in the vertical leg of the tube in which there was a return flow of the water toward the entrance of the tube.

(5) The insertion of a deflector in the model draft-tubes for the purpose of correcting the whirl component of flow was found to be impracticable.

Suggestions for the Design of Elbow Draft-Tubes.—

(6) The vertical leg of an elbow draft-tube should be trumpet-shaped. This shape is just as efficient as any other. It has the advantage over the conical tube that, for a given reduction in mean velocity in a given length of tube, the deceleration is less in the regions of higher velocity.

(7) The following suggestions are offered as a basis for designing the cross-sections of the bend of an elbow draft-tube: (*a*) At 0°, beginning of bend, use a circular cross-section; (*b*) at 22½°, use an elliptical section, with the major axis of the ellipse normal to the plane of the bend; (*c*) at 45°, use an ellipse of greater eccentricity for the inside than for the outside part of the section; and, (*d*) at the end of the bend, use a rectangular section.

(8) The radius of curvature of the inside walls of the bend of an elbow draft-tube should be smaller than the radius of curvature of the outside walls of the bend. The centers of the radii of curvature of the inside and outside walls of the bend should be located so that the distance between the walls diminishes from point to point along the axis of the bend.

(8) A comparison of the results of the tests on pipe bends and draft-tubes, as described in this paper, with the performance of the best elbow draft-tubes described in current technical literature, indicates that, similar to a conical or trumpet-shaped draft-tube of given length and flare in which there is an optimum ratio of the distance between a deflector plate and the bottom edge of the tube to the throat diameter, there probably is an optimum ratio of the distance between the top and bottom walls of the exit of the bend of an elbow draft-tube of given length and flare to the throat diameter of the tube. Further experimentation is needed to establish the optimum ratio to be used in design.

(10) When a guide-vane or "splitter" is used in the bend of an elbow draft-tube, it should extend to about the third point of the bend. The extension of a "splitter" completely around the bend effects no increase in efficiency for direct flow, and causes loss in efficiency for whirling flow. When a bearing partition is used in the lower leg of an elbow draft-tube, it should extend about half way around the bend to help divide the flow between the two halves of the tube when there is whirl component of the filamental velocities.

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APPENDIX

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

A NEW THEORY OF RAIL EXPANSION

By Alfred Africano,[1] Jun. Am. Soc. C. E.

Synopsis

The change in length of a rail resulting from a change in temperature is ordinarily taken as the product of the coefficient of linear expansion times the length times the change in temperature. The action of the ties in resisting longitudinal movement of the rail is neglected.

For a 33-ft rail, with ordinary track construction, the resulting error is less than 2%, and, therefore, may be justly neglected; for a 66-ft rail, however, it may be from 10% to 40%, depending upon how securely the rail is fastened to the ties; and for the long continuous welded rails, the error is so great that it is evident that tie resistance must be taken into account.

In the following analysis of the problem, formulas are developed which include the action of the ties, and, in addition, the effect of joint restraint. The resultant theoretical rail expansions are in good agreement with the general observations made on the German railroads, and the careful measurements made on four half-mile length rails in the Delaware and Hudson track, at Mechanicville, N. Y.

Notation.—The letter symbols in this paper are introduced in the text as they occur and are summarized for reference in Appendix I. An effort has been made to conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Materials"[2] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

The coefficient of linear expansion, n, for hard steel is equal to 0.0000073 in. per degree Fahrenheit per in. of length[3]. Therefore, the free expansion of a rail, expressed as a formula, is,

$$\Delta l_f = n \, \Delta t \, l \quad \text{......................... (1)}$$

Note.—Discussion on this paper will be closed in May, 1937, *Proceedings*.

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[2] A.S.A.—Z10*a*—1932.

[3] Bethlehem Manual of Steel Construction, Catalogue S-47, 1934, p. 299.

in which l = half the length of a rail; and Δl_f = change in length, due to a temperature change, Δt, when the rail is free to expand. If this expansion is prevented by fixing the ends of the rail, the resulting uniform stress, s, in the rail is, in pounds per square inch,

$$s = E\,\delta = E\,\frac{\Delta l_f}{l} = E\,n\,\Delta t \quad \text{....................(2)}$$

in which δ = unit strain, in inches per inch; and E = modulus of elasticity. For E = 30 000 000, $s = 219\,\Delta t$, or the stress is 219 lb per sq. in. for every degree change in temperature. It should be noted that this stress is independent of the length of the rail.

If $\Delta t = 100°$ F is the rise in temperature over that at which the rail was laid, the stress would be 21 900 lb per sq. in.; and if the rail section were a 100-lb A R A = B[4], with a cross-sectional area, A = 9.85 sq. in., the force, F,

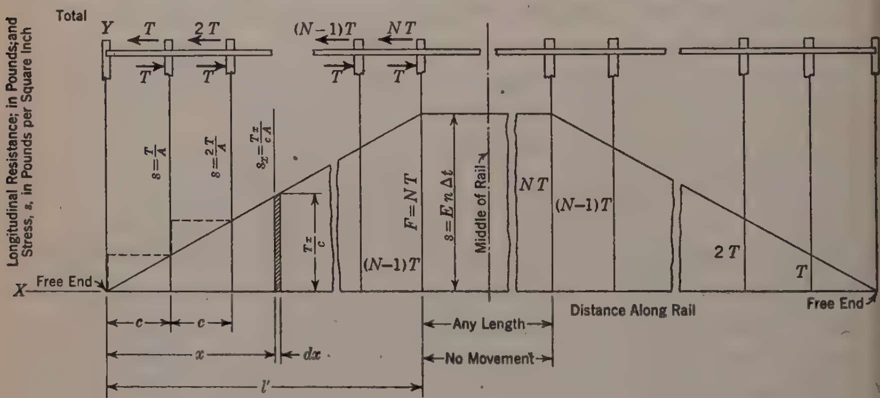


FIG. 1.—TEMPERATURE STRESSES IN A RAIL

required at each end to prevent expansion would be $F = sA$ = 21 900 × 9.85 = 216 000 lb. To develop this force in one-half the length, l, of a 33-ft rail, with the tie spacing, c, equal to 22 in., would require the average tie resistance to longitudinal movement, T, to be: $T = \frac{216\,000 \times 2 \times 22}{33 \times 12} = \frac{216\,000 \text{ lb}}{9 \text{ ties}}$ = 24 000 lb per tie.

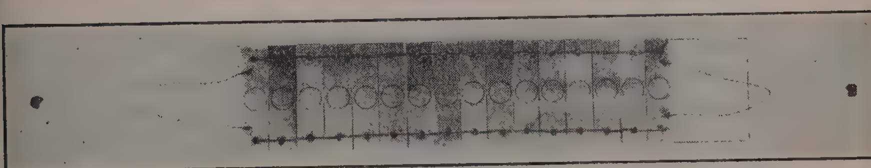
TABLE 1.—AVERAGE TIE RESISTANCE REQUIRED TO DEVELOP 216 000 POUNDS AT THE MIDDLE OF A RAIL

No. of 33-ft rails	Length, L, in feet	Resistance, T, in pounds per tie	No. of 33-ft rails	Length, L, in feet	Resistance, T, in pounds per tie
1.............	33	24 000	15.............	495	1 600
2.............	66	12 000	30.............	990	800
3.............	99	8 000	45.............	1 485	530
5.............	165	4 800	60.............	1 980	400

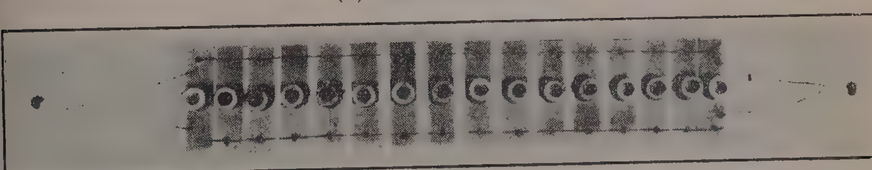
[4] Bethlehem Manual of Steel Construction, Catalogue S-47, 1934, p. 78.

Since it is improbable that one rail-fastening and the tie will develop a resistance of 24 000 lb, unless set in concrete, a 33-ft rail expands quite freely; but when several rails are made continuous the average tie resistance drops to an amount that can be developed reasonably, and the fixed-end condition is approached. Table 1 illustrates how the average tie resistance required to prevent expansion varies inversely as the length of the rail.

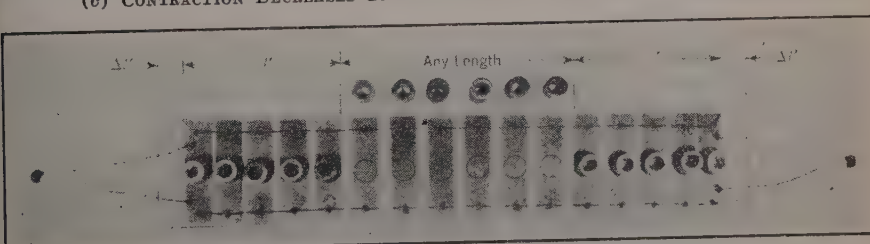
(*a*) ORIGINAL LENGTH OF RAIL



(*b*) FREE CONTRACTION



(*c*) CONTRACTION DECREASED BY LONGITUDINAL RESISTANCE, NOT UNIFORM



(*d*) NO LONGITUDINAL RESISTANCE REQUIRED OF ENDS IN THE FIXED PART OF THE RAIL

FIG. 2.—MODEL SHOWING THAT RESISTANCE TO LONGITUDINAL MOVEMENT DEVELOPED BY THE RAIL FASTENINGS NEAR THE ENDS CAN BE SUFFICIENT TO PREVENT CONTRACTION OF THE REMAINDER OF THE RAIL.

The free ends of the continuous rail will always expand a certain extent. To determine this actual expansion for any length of rail, and for any degree of tie resistance, it is simply necessary to determine how much expansion is prevented, and to subtract that from the free expansion. The concept of the increasing resistance to longitudinal movement, from zero at the free ends to a maximum when N ties completely fix the rail, is illustrated in Fig. 1.

It may be demonstrated very effectively with the model shown in Fig. 2, which consists of two elastic bands fastened to cardboard strips which rep-

resent equally spaced sections of the rail base. By first stretching the elastics, then placing weights on the strips to obtain various degrees of longitudinal resistance (rail-fastenings), and, finally, releasing the ends, the action will closely approximate that of a rail following a drop in temperature. Using compressed springs instead of the elastic bands would illustrate the case in expansion.

Dervation of Formulas for Expansion (or Contraction)

Without Joint Restraint.—For simplicity consider the tie resistance to be distributed uniformly along the rail so that $\frac{T}{c}$ is in pounds per linear inch. (A solution by summation of the terms of the arithmetical progression representing the expansion that is prevented in consecutive tie spans shows the error of this assumption to be small, and quite negligible for long rails.) The compressive stress at any point, x in. from the end of the rail, resulting when the tie resistance prevents expansion with a rise in temperature (or tensile stress with a drop in temperature), is, from Fig. 1:

$$s_x = \frac{T\,x}{c\,A} \qquad (3)$$

and may be considered constant in the differential element, dx.

Hooke's law states that this stress is proportional to the strain; and this is true no matter how that strain is produced, whether by a load in the ordinary case with no change in temperature, or, as in this case, by suppressing part of the normal change in length required to leave the rail unstressed at the different temperature. Therefore, the strain in the differential element produced by preventing its lengthening or shortening an amount $d(\Delta l)$ can be substituted in the fundamental formula (Equation (3)) and equated to the stress in the rail at that point, as follows:

$$s_x = E\,\delta_x = E\frac{d\,(\Delta\,l)}{dx} = \frac{T\,x}{c\,A} \qquad (4)$$

Solving for the change in length that is prevented,

$$d\,(\Delta\,l) = \frac{T\,x\,dx}{c\,A\,E} \qquad (5)$$

The total expansion (or contraction), Δl_r, that is prevented in the half-length, l, is now found by integrating between the limits, $x = 0$ and $x = l$, or,

$$\Delta\,l_r = \int_0^l \frac{Tx\,dx}{c\,A\,E} = \frac{T\,l^2}{2\,c\,A\,E} \qquad (6)$$

(This result of integrating Equation (6) can be obtained without the use of calculus by noting that the sum of all the suppressed unit changes in length is equal to the area of the triangle which represents them as

changing uniformly from zero at the free end to a maximum when the rail is fixed. However, it is best to use calculus to illustrate the general method to be used in case the resistance to longitudinal movement of the rail is found to be some empirical curve rather than the simple straight-line variation assumed in this analysis.) The net expansion is the free expansion minus that which is prevented, or,

$$\Delta l = \Delta l_f - \Delta l_r = n \Delta t\, l - \frac{T\, l^2}{2\, c\, A\, E} \quad \text{.............(7)}$$

The maximum end expansion occurs when l becomes equal to the critical length,

$$l' = \frac{E\, n\, \Delta t\, A\, c}{T} \quad \text{.......................(8)}$$

required to develop the longitudinal force, $F = E\ n\ \Delta t\ A$, which fixes the rail thereafter, so that there is no further increment in the total expansion. If this is not sufficiently obvious from Fig. 1, a convincing proof may be obtained by setting the first derivative of Equation (7) equal to zero for a maximum, and solving for l. Consequently, the tie resistance, T (which is simply a frictional reaction opposing any movement), does not come into action at all in any additional length of rail placed between the critical lengths, l', at each end, as the model action shown in Fig. 2(*d*) clearly demonstrates.

Substituting the value of this critical length in Equation (7) results in the following simple expression for the maximum end movement of a rail without joint restraint:

$$\Delta l'_{max.} = \frac{E\, n^2\, (\Delta t)^2\, A\, c}{2\, T} = \frac{\Delta l'_f}{2} \quad \text{.................(9)}$$

Stated as a theorem:

Theorem 1.—For free ends, the maximum expansion at each end of a long continuous welded rail is one-half of what the free expansion would be in the length that is required to fix the rail.

That this is what should be expected is evident when it is considered that the expansion varies from its normal unit free expansion at the end of the rail, uniformly, down to zero at the point along the rail where the force (independent of the length) required to fix it becomes fully developed.

Note that Equation (9) shows that the maximum expansion: (1) Is independent of the length of the rail; (2) varies as the square of the temperature difference for a given track; and, (3) for different tracks, varies inversely as the tie resistance. This is quite a departure from the time-honored method of computing allowable gaps for 33-ft rails. That the actual expansion follows Equation (9) in principle is strongly supported by the careful observations made on four continuous rails, each about 2 600 ft long, at Mechanicville, and the general observations made in Germany and other countries where long rails have been installed.

Example 1.—For the case in which $n = 0.0000073$, $\Delta t = 100°$ F; $c = 22$ in.; $T = 1\,000$ lb per tie; $A = 9.85$ sq in.; and $E = 30\,000\,000$ lb per sq in.: $F = 216\,000$ lb; $N = 216$ ties; and $\Delta l'_{max} = 1.74$ in. This maximum expansion would occur at each end of a continuous rail, $2l' = \frac{216 \times 22 \times 2}{12} = 792$ ft long. For any longer rail, the additional length would be fixed completely against expansion, so that the total expansion at the ends would remain the same.

Expansion of Rails Shorter Than the Critical Length.—For lengths less than 792 ft, the net expansion can be derived from Equation (7), which reduces to,

$$\Delta l = 0.00073\, l - (0.077 \times 10^{-6})\, l^2 \quad \text{(10)}$$

for this case.

The theoretical expansions from this equation for various lengths are given in Table 2 to show what can be expected under the assumed conditions.

TABLE 2.—THEORETICAL EXPANSION AS COMPUTED BY EQUATION (10)

Number of 33-ft. rails	Length, L, in feet	Number of ties, N (end to middle of rail)	Δl_t, in inches	Δl_r, in inches	Net expansion, Δl, in inches	Number of 33-ft rails	Length, L, in feet	Number of ties, N (end to middle of rail)	Δl_t, in inches	Δl_r, in inches	Net expansion, Δl, in inches
1.....	33	9	0.145	0.003	0.14	10...	330	90	1.45	0.30	1.15
2.....	66	18	0.29	0.012	0.28	20...	660	180	2.90	1.21	1.69
4.....	132	36	0.58	0.05	0.53	24...	792	216	3.48	1.74	1.74*
6.....	198	54	0.87	0.11	0.76	30...	990	270	3.48	1.74	1.74

* $\Delta l'_{max}$.

The maximum expansion varies inversely as T. Therefore, a value of $T = 5\,000$ lb (claimed possible in certain types of track construction), instead of the $T = 1\,000$ assumed, would give a result only one-fifth as great (that is, 0.35 in.) and $N = 43$ would be the number of ties required on each side of the middle of the rail to prevent further expansion for longer rail lengths. For this case the critical length, l', would be 79 ft.

There probably would be a slight movement of the tie proportional to the resistance for so high a value of T. To show a possible cause of discrepancy in the observed and theoretical rail expansions, as described in this paper, and to show also that it is probably negligible if H, the modulus of horizontal resistance, is about the same as u, the vertical modulus of foundation[5], the following calculation is given. (The modulus of foundation, u, is the load per unit length of the rail required to produce a vertical deflection of the foundation equal to unity. By analogy, the modulus of horizontal resistance, H, is the force per unit length of the rail required to produce a

[5] Introduced in the First Progress Report of the Special Committee on Stresses in Railroad Track, *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1191.

horizontal deflection of the foundation equal to unity.) If, on the average, each tie moved about the same distance to develop its value of tie resistance, T, the additional movement of the end of the rail would be:

$$\Delta l_H = \frac{T}{c\,H} \quad \text{(11)}$$

Substituting $T = 5\,000$, $c = 22$ in., and $H = u = 1\,500$ lb per in. per in. (an average value), in Equation (11), the additional movement is found to be only 0.15 in. The total expansion at the end is, then, 0.35 + 0.15 = 0.50 in., for this case.

A length of continuous rail, 200, 400, or even 1 000 ft long, would give the same theoretical expansion (0.50 in.), that is calculated for this 158-ft length. It follows that, for preventing expansion of the remaining length, only 79 ft at each end of the long continuous welded rail need have the higher resisting and more expensive rigid type of track. Maintenance-of-way engineers contemplating the change to welded joints will be interested in verifying this conclusion. In this connection the writer wishes to make it clear that, although he recognizes the desirability of rigid track for the entire length of the rail, his present purpose is to emphasize and call attention to the basic fact that the longitudinal resistance is developed only as a reaction to movement, and this occurs near the ends of the rail, not at the middle, fixed part.

For rail laid in cold weather, it may even be necessary to provide lateral anchors, in addition to stiff track, to prevent the rail from buckling laterally and from being thrown out of alignment. On the other hand, for tangent rail laid in hot weather, with only the tensile stresses resulting from a drop in temperature, no special type of track may be necessary.

Formulas Including Joint Restraint

Fig. 3. shows the longitudinal resistance assumed to be increasing uniformly from the initial value, P, representing the joint restraint, to the maximum value, $F = E\, n\, \Delta t\, A$, that is required to fix the rail. As in the case without joint restraint, the change in length that is prevented depends directly on the stress developed in the rail, which is (see Equation (3)):

$$s_x = \frac{P}{A} + \frac{T\,x}{c\,A} \quad \text{(12)}$$

but (see Equation (4)):

$$s_x = E\,\delta = E\,\frac{d\,(\Delta l_r)}{dx} \quad \text{(13)}$$

Therefore,

$$\Delta l_r = \int_0^x \frac{1}{E}\left(\frac{P}{A} + \frac{T\,x}{c\,A}\right) dx \quad \text{(14)}$$

and,

$$\Delta l_r = \frac{P\,x}{A\,E} + \frac{T\,x^2}{2\,c\,A\,E} \quad \text{(15)}$$

The net expansion for rails shorter than twice the critical length, l'' $\left(\text{that is, } l < \frac{c\,(F-P)}{T}\right)$ is:

$$\Delta l = n\,\Delta t\,l - \frac{P\,l}{A\,E} - \frac{T\,l^2}{2\,c\,A\,E} \quad \ldots\ldots\ldots\ldots\ldots\ldots(16)$$

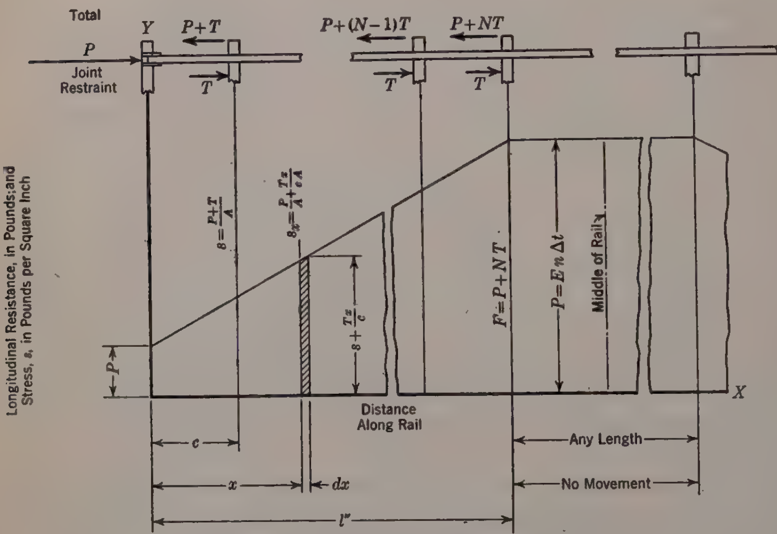


Fig. 3.—Forces in a Rail with Restrained Joints.

Substituting the critical length,

$$l'' = \frac{c\,(E\,n\,\Delta t\,A - P)}{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

required to finish fixing the rail (see Equation (8)), into Equation (16) will result in the simplified expression for maximum end movement with joint restraint, independent of the length, for any length of rail greater than the critical length (see Equation (9)):

$$\Delta l''_{\text{max.}} = \frac{c\,(E\,n\,\Delta t\,A - P)^2}{2\,T\,A\,E} = \frac{c\,(F-P)^2}{2\,T\,A\,E} \quad \ldots\ldots\ldots\ldots(18)$$

To Determine Values of P and T by Observation of End Expansions.— The procedure for determining the values of P and T by observing end expansions, is as follows:

(1) Measure Δl_1, the end movement with joint restraint from a fixed reference point, for a known rail temperature change from the laying temperature.

(2) Remove joint bolts, strike joint bars with a hammer to loosen their grip completely so that the ideal condition of free ends will be realized—to warrant applying Equation (9), which was derived with this assumption. Now, measure Δl_2, the end movement with free ends.

(3) Substitute Δl_2 and the temperature difference in Equation (9) (or Equation (18) which reduces to Equation (9) when $P = 0$) and solve for the correct tie resistance, T_2.

(4) Placing this correct value, T_2, and the first measured end movement, Δl_1, with the same temperature difference, in Equation (18), solve for the joint restraint, P; thus:

$$P = F - \sqrt{\frac{2\,(\Delta l_1)\,T_2\,A\,E}{c}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Equation (19) can be simplified so that the ratio of the two measured end movements can be substituted directly in the formula to give the ratio of joint restraint to the total force required to fix the rail. Multiplying the term under the radical by $\frac{\Delta l_2}{\Delta l_2}$, or its equivalent, $\dfrac{\frac{E\,n^2\,(\Delta t)^2\,A\,c}{2\,T_2}}{\Delta l_2}$, from Equation (9),

$$P = F\left(1 - \sqrt{\frac{\Delta l_1}{\Delta l_2}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Comparison with Observed Expansions

General.—For a comparison with general observed expansions in Germany, attention is called to the following quotations[6];

"In Germany a 1 000′ length of rail moved $\frac{1}{4}$″ and $\frac{3}{8}$″ at the joints. * * * The German engineers say the amount of movement at the ends regardless of the length is apparently only the normal movement for 40 ft. rails." [The movements for 100, 200, and 400-ft lengths were compared.]

In Australia, engineers of the Victorian Railways found that the actual expansion and contraction was about one-half the "theoretical" expansion. This was for rails as long as 250 ft, with standard track construction.

Graphical Solution

Equation (9) can be transposed and written:

$$T = \frac{K\,(\Delta t)^2}{\Delta l'_{\text{max.}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which,

$$K = \frac{E\,n^2\,A\,c}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Example 2.—Equation (21) can be solved by reference to Fig. 4. For example, let the observed end movement, $\Delta l' = \frac{5}{16}$ in.; $\Delta t = 58°$ F; $E = 30 \times 10^6$; $n = 7 \times 10^{-6}$; $c = 22$ in.; $A = 12.86$ sq in. By Equation (22) the value of the track constant, K, is found to be 0.20. Then,

[6] "Thermit Rail Welding," in a brochure pub. by The Metal and Thermit Corporation, New York, N. Y.

entering Fig. 4 at $\Delta t = 58°$ F, intersect the curve, $K = 0.20$; proceed horizontally from the intersection to the radial line, $\Delta l' = \frac{5}{16}$ in., to determine the tie resistance, $T = 2\,100$ lb per tie.

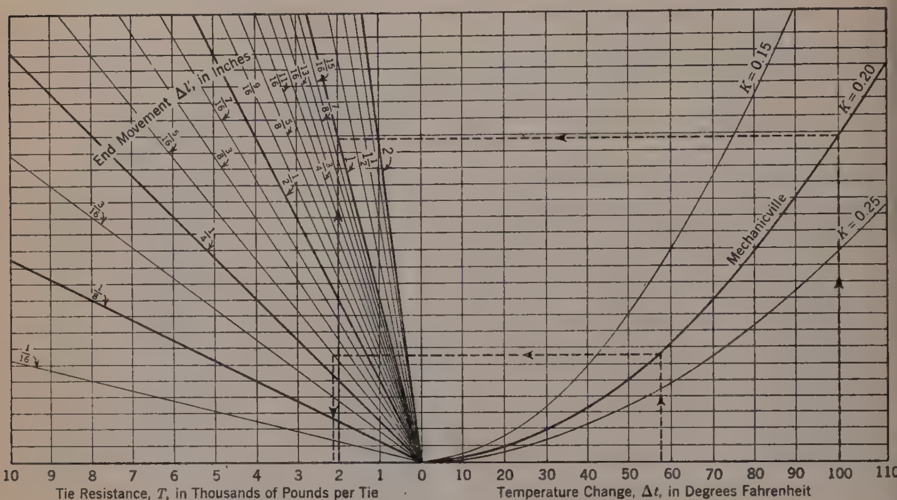


FIG. 4.—GRAPHICAL SOLUTION OF EQUATION (9).

Example 3.—Varying the conditions in Example 2, let $T = 2\,000$ lb per tie; $\Delta t = 100°$ F; and, $K = 0.20$. Proceed vertically from $\Delta t = 100$ to $K = 0.20$, as before; continue horizontally from this intersection to an intersection with the vertical line, $T = 2\,000$; and read the answer, $\Delta l' = 1$ in. on the diagonal lines.

APPLICATION OF THE FORMULAS TO THE MECHANICVILLE, N. Y., OBSERVATIONS

Example 4.—Four continuous half-mile lengths of welded A.R.E.A., 131-lb rail were laid in the main tracks of the Delaware and Hudson Railroad, at Mechanicville, on an extremely hot day, and the closing rail temperature was noted. Subsequently, the maximum observed contraction at one end was 0.75 in. for a temperature drop of 82° F. If the joint restraint were slight, or were removed entirely, Equation (9) (for free ends) applies.

The values of the track constants are: Cross-sectional area of rail, $A = 12.86$ sq in.; $E = 30 \times 10^6$; $n = 7 \times 10^{-6}$; and $c = 22$ in.

Substituting the observed values of $\Delta l_{max.}$ and Δt in Equation (9) (see heading "Graphical Solution") and neglecting the small tie movements gives a value for the average tie resistance to longitudinal movement of $T = 1\,860$ lb per tie. The average calculated value of T from forty observations (that is, both ends of four rails measured at five different times after installation) was 2 100 for an average movement of $\frac{5}{16}$ in., and an average temperature drop of 58° F.

Using a value of, say 2 000 for T, the maximum contraction for a temperature drop of 100° can now be predicted from Equation (9) (as shown in

Example 3), and results in the value, $\Delta l = 1$ in. This will be the same for any rail (with similar track constants) longer than twice the critical length, and from Equation (9) this length is: $L = 2l' = \dfrac{4 \Delta l'_{max.}}{n \Delta t} = 475$ ft. Thus, the actual contraction is only one-half of what the free contraction would be at each end in a 475-ft length of rail, and one-eleventh of what it would be at each end in the 2 600-ft length.

Example 5.—With Joint Restraint.—Since it is quite likely that joint restraint did exist when the observations were made at Mechanicville, it is important to show how the additional data obtained in the suggested procedure for determining the degree of the joint restraint is to be applied in Equations (18) and (20).

Suppose after Δl_1 had been measured equal to 0.75 in., the joint bolts had been removed to allow the end to move freely an additional 0.50 in., making $\Delta l_2 = 1.25$ in. The correct value of the tie resistance can now be solved from Equation (9), or more directly by making use of the fact that the maximum end movement for free ends varies inversely as the tie resistance:

$$\frac{\Delta l_1}{\Delta l_2} = \frac{T_2}{T_1} \quad \text{..........................(23)}$$

or, $T_2 = \dfrac{\Delta l_1}{\Delta l_2} T_1 = \dfrac{0.75}{1.25} \times 1\,860 = 1\,120$ lb per tie. The number of ties required to develop the force, $F = E\ n\ \Delta t\ A = 222\,000$ lb, would be, for free ends: $N_2 = \dfrac{222\,000}{1\,120} = 198$ ties, and the critical length, $l' = 198 \times \dfrac{22}{12}$ $= 364$ ft.

The number of ties and the length of rail that were actually in use, however, depend upon the degree of joint restraint. Applying Equation (20): $P = 222\,000 \left(1 - \sqrt{\dfrac{0.75}{1.25}}\right) = 50\,000$ lb. The number of ties in use with this degree of restraint was: $N_1 = \dfrac{222\,000 - 50\,000}{1\,120} = 154$ ties; and, the critical length: $l'' = N_1 C = 154 \times \dfrac{22}{12} = 282$ ft. This means that the maximum contraction (0.75 in.) would have remained the same at each end, for any length of welded rail from 564 ft to the actual length of the installation, 2 600 ft.

If the rail had been shorter than twice the critical length required to fix it completely, this fact would have been evident, according to Theorem 1, by its observed free end movement being greater than one-half the calculated free contraction. Equations (7) and (16) would then apply.

Conclusion

A complete analysis of the stresses in welded rail would have to include the complicated wheel load stresses in addition to the temperature stresses

described in this paper. The various ways in which a rail can be stressed are summarized in Appendix II to show the extent of this subject and the necessity for limiting the present investigation to one particular aspect.

The principal objection to the introduction of continuous welded rail has been the fear of excessive expansion or contraction. Therefore, the writer's purpose has been solely to present a theortical explanation of the small observed expansions in existing long rails which are not accounted for under the existing "theory."

It is hoped that, with more observations of expansions, the correctness of the formulas presented in this paper will become established. It is only with this knowledge of what fundamentally takes place at the ends of the rail that the proper approach can be made to the economies resulting from the elimination of rail joints.

Acknowledgments

The writer expresses his appreciation to his superior, F. W. Gardiner, M. Am. Soc. C. E., for making available the data suggesting the problem, and for the discussions which served as the basis for this paper. Credit is due G. M. Magee, Assistant Engineer, Kansas City Southern Railway Company, for his valuable help in checking the mathematical derivation of the formulas, and for his suggestions.

APPENDIX I

Notation

The following letter symbols have been adopted for use in the paper:

A = cross-sectional area of a rail, in square inches;
c = distance between ties; tie-spacing, in inches;
E = modulus of elasticity;
F = force; the total force required to restrain a rail, in pounds;
f = a subscript denoting "free expansion";
H = modulus of horizontal resistance;
K = a track constant (see Equation (22));
L = total length of a rail or track;
l = length of a rail from one end to the middle, when the longitudinal resistance is insufficient to develop a force, F, to restrain a rail; Δl_f = change in length due to temperature change (in inches), with free expansion in the length, l; Δl_r = change in length due to temperature change, with the rail restrained against expansion; l' = critical length, or length required to develop a restraining force of F lb, or a force sufficient to restrain the rail entirely; l'' = a critical length which includes the effect of joint restraint; l_1 = end movement when a joint is restrained, measured from a fixed reference point and corresponding to a known value of Δt; Δl_2 = end movement when a rail is permitted to expand freely;

N = number of ties required to develop a force, F;
n = coefficient of linear expansion, in inches per degree Fahrenheit per inch;
P = initial longitudinal resistance to expansion, or contraction;
r = a subscript denoting "resists expansion";
s = unit stress;
T = resistance at the ends of each tie, in pounds; T_2 = correct tie resistance;
t = temperature, in degrees Fahrenheit; Δt = a change in temperature;
u = vertical modulus of foundation; load, in pounds per linear inch of rail per inch of vertical deflection;
δ = unit strain resulting from a stress, s.

APPENDIX II

Wheel Load Stresses

Wheel loads may contribute to the stresses in rails in the following ways:

1.—Static Stresses.—The rail is considered a loaded structure on an elastic foundation. Under this assumption corroborated by field tests with extensometers, the stress, s_x, in the extreme fiber at a distance $\pm x$ from the position of the wheel load on the rail, is:

$$s_x = \frac{P}{4\,\beta\,Z}\,e^{-\beta x}\,(\cos\beta\,x - \sin\beta\,x) \qquad (24)[7]$$

in which P is the wheel load, in pounds; β is a symbol representing a quantity, $\sqrt[4]{\frac{u}{4\,E\,I}}$; I and Z are, respectively, the moment of inertia and the section modulus of the rail about the horizontal neutral axis; u is the modulus of foundation; E is the modulus of elasticity for steel; and e is the base of the natural logarithms, or 2.718.

2.—Dynamic Stresses[8].—These are impact stresses caused by low spots in the rail, flat or out-of-round areas on the wheel, rail-joint gaps, differences in the elevation of adjoining rails, and the vertical components of the connecting-rod and counterbalance forces in the case of steam locomotives.

3.—Shear Stresses.—Formulas developed by Thomas and Hoersch[9] give values for the high localized shear stresses that form slightly below the contact surface of the rail and wheel. Once the initial crack starts from this

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[7] For derivation of formula see "Applied Elasticity," by S. P. Timoshenko and J. M. Lessells, 1928, p. 141.

[8] "Stresses in Railroad Track" by S. P. Timoshenko and B. F. Langer, *Transactions*, A. S. M. E., November 30, 1931; also, "Rail Stresses and Locomotive Tracking Characteristics Found in Tests on the Great Northern Railway," by J. P. Shamberger and B. F. Langer, *Bulletin 339*, A. R. E. A., September, 1931.

[9] "Stresses Due to the Pressure of One Elastic Solid Upon Another," by H. R. Thomas and V. A. Hoersch, *Bulletin 212*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1930.

cause, it spreads under the constant reversal of bending stresses of much smaller magnitude. Transverse fissures are examples of this type of overstress in the rail.

4.—Web, Fillet, and Bolt-Hole Stresses.—The sudden change of cross-section causes high concentration of stress which may result in minute initial cracks. These stresses can be calculated by photo-elastic methods.

5.—Torsion Stresses.—These stresses may be caused by eccentric vertical loads, and the lateral forces due to swaying, unbalanced centrifugal and superelevation forces, and flange pressures.

6.—Vibration Stresses.—Vertical and lateral oscillations of the truck and rail may produce momentary high stresses.

In addition to the specific references given in the footnotes, data on rail stresses may be found in the Progress Reports of the Special Committee on Stresses in Railroad Track published in the *Transactions* of the Society; the *Proceedings* of the American Railway Engineering Association; and the quarterly reports of the Rails Investigation being undertaken at the Engineering Experiment Station, University of Illinois, at Urbana, Ill.

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PAPERS

ECONOMICS OF HIGHWAY-BRIDGE FLOORINGS OF VARIOUS UNIT WEIGHTS

BY J. A. L. WADDELL,[1] HON. M. AM. SOC. C. E.

SYNOPSIS

The comparative economics of any two or more types of modern highway-bridge floors is presented in this paper. The method provides a means of answering questions of relative economy in a very few minutes, beyond the peradventure of doubt in regard to accuracy, either to-day or at any future time. The following information concerning each competing type must be known: The thickness of the flooring and its weight per square foot; the weight of metal per square foot of floor, between the stringers and the flooring with a stringer spacing of 5 ft, if there is any such special metal (or, otherwise, the extra weight of metal per square foot, in the floor system, due to a closer spacing of stringers); and the cost per square foot of flooring in place.

Figs. 1 to 8, with Table 1, afford all the information required for making such economic comparisons; and the text describes the conditions that governed the computations for all diagrams and tables. Invariably, both superstructure and substructure costs are considered, under the average or the usual conditions that were assumed for the latter.

Certain unit prices of materials in place were adopted for making the calculations of total costs per linear foot for the more than two hundred and fifty spans that were specially computed in making the investigation. Due cognizance was taken of the fact that the unit prices of both superstructure and substructure material, in place, increase as the total quantities thereof diminish; and certain empirical formulas are furnished for the proper and adequate correction (for this reason) of all such unit prices.

The bridges covered in the records are for simple-truss and cantilever spans, for both wide and narrow roadways, and for both carbon-steel and sili-

NOTE.—Discussion on this paper will be closed in May, 1937, *Proceedings*.

[1] Cons. Engr., New York, N. Y.

con-steel structures. Attention was paid, wherever necessary, to the augmenting of total metal weights due to wind loads, and to the effect on costs of both superstructure and substructure from the spreading of trusses beyond the requirement for width of roadway. The curves were determined upon the basis of there being solid rock or an equivalent reasonably hard foundation for piers; and the effect on the comparative economies of using pile foundations has been treated.

Full explanations are provided regarding the utilization of the diagrams and tables; and three practical cases for bridge-floor competitions have been solved in the text. The economic effects of using light floorings on vertical-lift and bascule spans and on suspension bridges have been treated at some length. The entire investigation has been made upon an absolutely unbiased basis; and the method of presenting the cost diagrams represents the acme of impartiality.

Introduction

In addition to the usual type of reinforced concrete flooring for modern highway bridges, termed herein the "Standard", there are a number of patented floorings, the main object of which is to reduce the dead load on the structure and thus save metal and, therefore, expense. Because these special types are more costly than the ordinary type of reinforced concrete flooring, and, in addition, involve royalties, they are likely to increase rather than to decrease the total cost of structure in short-span bridges.

Other considerations being equal (that is, ignoring all claims for superiority of surface, resistance to skidding, absence of pitch or crown, better drainage, non-collection of snow and ice, etc.), the bridge engineer, in the interests of his clients, will very properly desire to design his structures so as to keep their total cost down to a minimum by selecting the most economic type of flooring. Each patent owner naturally wants people to believe that his special type is not only superior to all other types, but is also the least expensive, when the total cost of the completed structure is in question. Comparing any two types of equal suitability, it is evident that the lighter the flooring the smaller will be the percentage of steel required for both the floor system and the trusses; but the total saving of metal may not be great enough to offset the difference in the costs of flooring for the two types under comparison, after the royalties have been included.

It is evident that the longer the span the greater is the proportionate saving in metal weight of trusses resulting from the reduced dead load. As far as the different types of floor construction are concerned, it is almost axiomatic that the lighter the flooring the greater, generally, is (or should be) its actual cost per unit of area, because the thinner it is the larger must be its volume of expensive steel and the smaller its volume of cheap concrete. This statement does not always apply to varying thicknesses of flooring of the same type.

Since, at present (1937), few, if any, bridge specialists are at all certain about the comparative ultimate economics of the various bridge floorings, the establishment of an absolutely correct and impartial method for set-

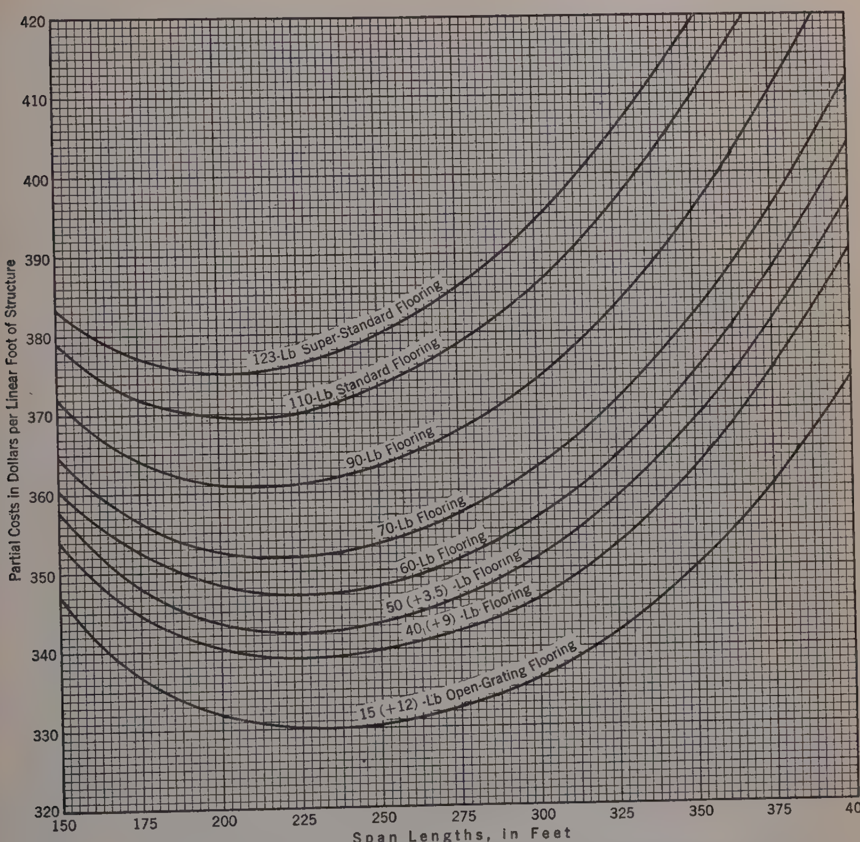


FIG. 1.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF CARBON STEEL; 45-FOOT CLEAR ROADWAY

tling the question would be a real boon, not only to the Engineering Profession, but also to the promoters of bridge projects and to the financiers whose money is to be spent for the materialization of the desired structures. It is for that reason that this paper, with Figs. 1 to 8, has been prepared.

Basic Assumptions

A series of solid floorings having weights per square foot of floor equal to 123, 110, 90, 70, 60, 50, and 40 lb, as well as an open-grate flooring computed for 15 lb per sq ft, plus an allowance of 12 lb per sq ft for small I-beam cross-girders, was assumed; and, for each one of the series, an arbitrary (but logical) cost per square foot was adopted, including royalty, these costs generally increasing as the weight of floor-slab diminishes.

Next, cost curves were plotted to cover the substructure, the superstructure metal work contained in the floor system proper (on the basis of 5-ft stringer spacing), the lateral system, and the trusses (excluding the metal on piers and in the anchorages) for both simple-truss spans and cantilever bridges, in both carbon steel and silicon steel, and for both wide roadways and narrow roadways. These curves do not contain the costs per linear foot of the flooring, the metal in extra stringers, or that between flooring and floor systems, the guard-rails, or the hand-rails; but the costs of these enumerated items (except only the hand-rails) are given in tables, using the assumed unit prices that are adopted in the text for this investigation. In the future, the foregoing costs will need to be computed for each special case as it arises before the diagrams can be utilized. This program, at first reading, may appear a bit complicated; but the examples, given hereinafter, for utilizing the cost curves will make the procedure perfectly clear.

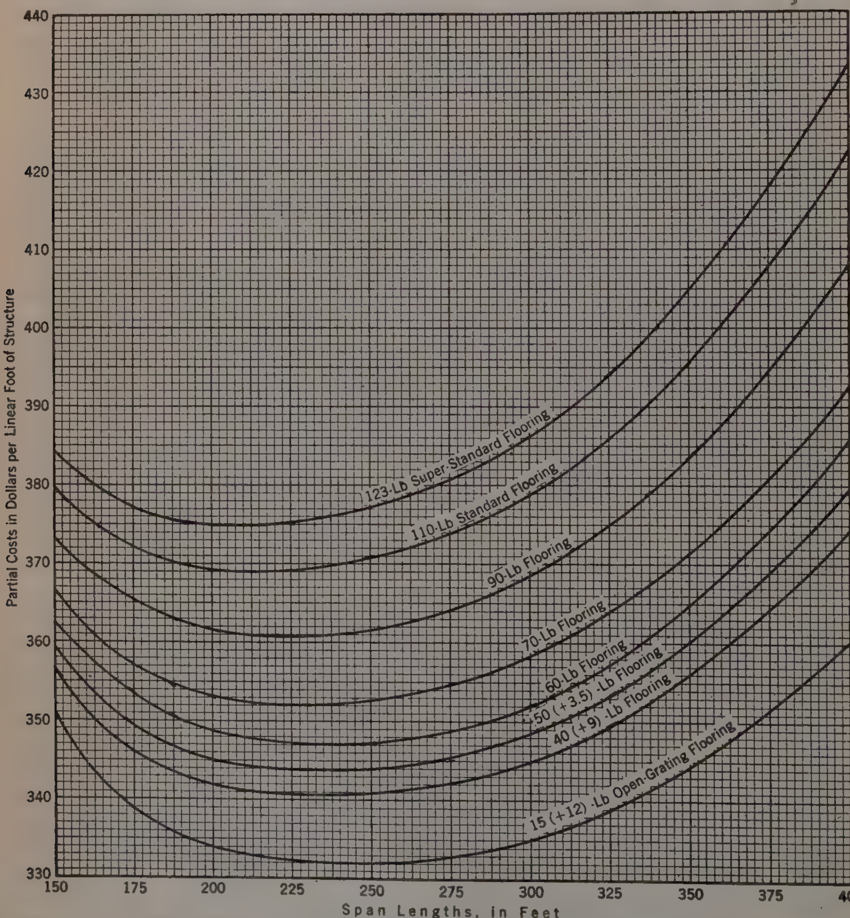


FIG. 2.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF SILICON STEEL; 45-FOOT CLEAR ROADWAY; SPANS, 150 FEET TO 400 FEET.

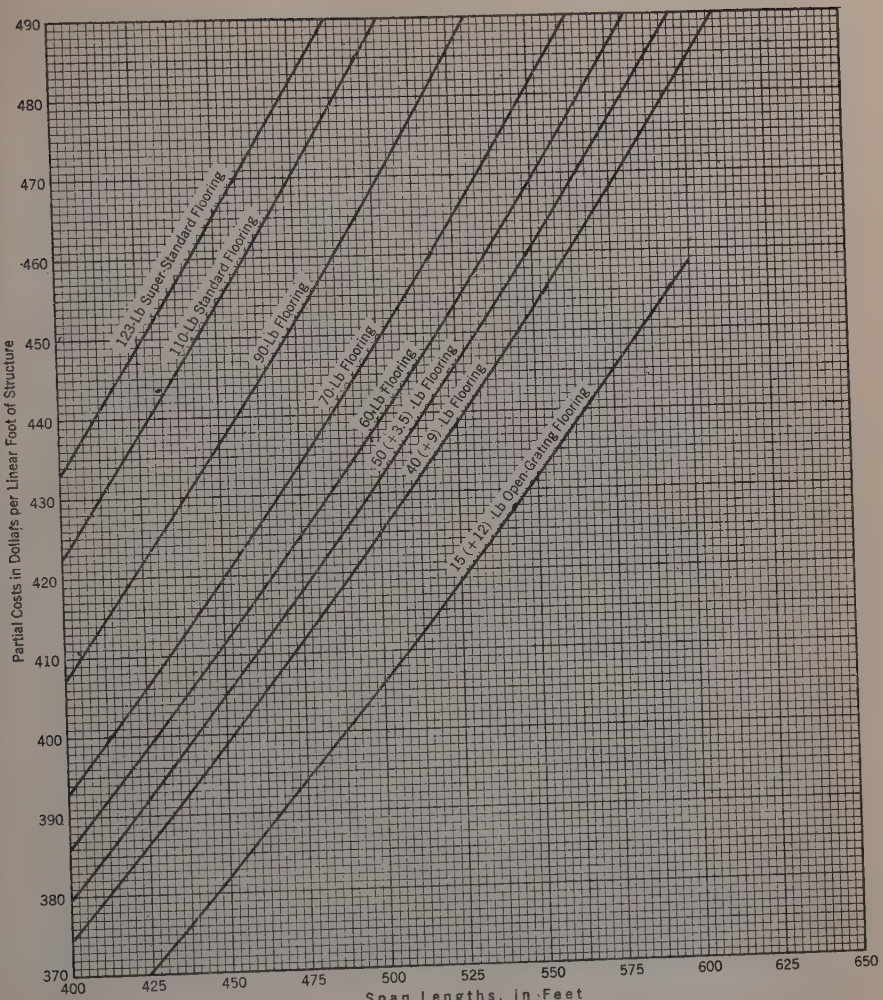


FIG. 3.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF SILICON STEEL; 45-FOOT CLEAR ROADWAY; SPANS, 400 FEET TO 650 FEET.

Because of its common use, a 110-lb, 9-in. slab floor of ordinary reinforced concrete, consisting of an 8-in. structural slab with its thickness increased 1 in. for wear, has been selected as the "Standard" type of non-patented flooring. The 123-lb flooring, designated as "Super-Standard", consists of an 8-in., reinforced concrete slab, covered with a 2-in. thickness of asphalt paving. For both these types, the stringer spacing was assumed to be 5 ft.

The 90-lb flooring represents a 7-in., concrete slab reinforced with trusses of welded-bar or other rigid construction, also carried by stringers spaced 5 ft on centers. This thickness includes 1 in. of wearing surface.

Below a certain limit for the special floorings, it is necessary to reduce the stringer spacing as the slab thickness decreases; hence, a proper varying increment to the floor-system weight was made for a number of the assumed reference types.

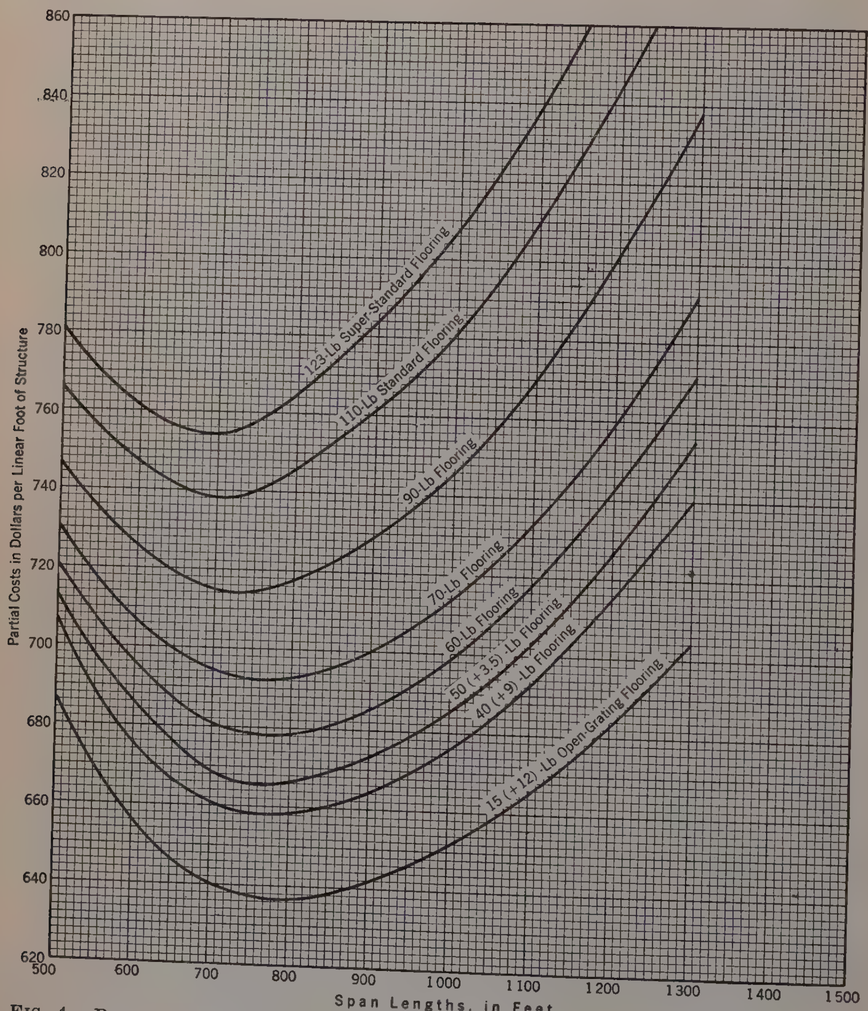


FIG. 4.—PARTIAL UNIT COST OF TYPE *A*, CANTILEVER HIGHWAY BRIDGES OF SILICON STEEL; 45-FOOT CLEAR ROADWAY.

The open-grate flooring of this comparative series was based on using $\frac{3}{16}$-in. metal throughout, but it is better practice to make the straight (or carrying bars $\frac{1}{4}$ in. thick. This would add 2 lb per sq ft to the weight of the floor, but would permit a saving of 1 lb per sq ft (through slightly wider spacing) in the weight of the small transverse I-beams, making a net increase of 1 lb of metal per sq ft. Consequently, if the heavier carrying bars are used, it will be necessary to allow a rebate of 4 cts per sq ft of floor, but,

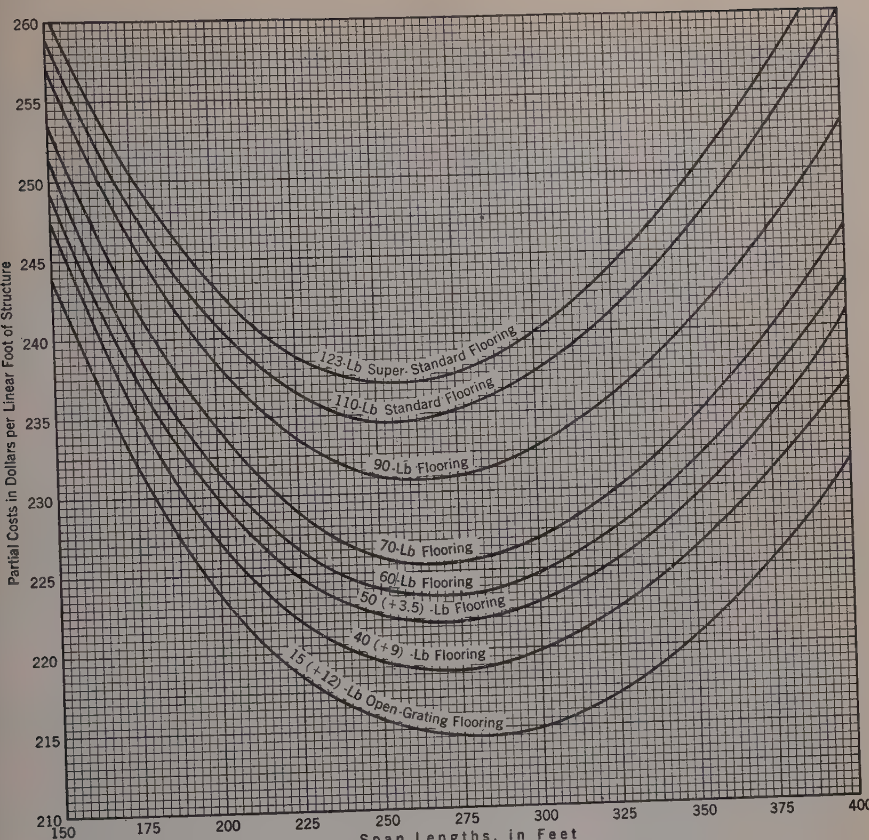


FIG. 5.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF CARBON STEEL; 20-FOOT CLEAR ROADWAY.

on the other hand, to provide for the increased cost of the structure due to the extra load of 1 lb per sq ft. In Table 1 are recorded full data concerning these eight comparable floorings.

TABLE 1.—DATA CONCERNING COMPARABLE FLOOR TYPES

Item No.	Type	Total weight of floor, in pounds per square foot	Cost in place per square foot	Allowance included in cost for extra floor metal as compared with the standard, Item No. 2
1	Super-standard flooring.....	123	$1.00	None
2	Standard flooring..........	110	0.80	None
3	90-lb flooring...	90	1.00	None
4	70-lb flooring...	70	1.50	None
5	60-lb flooring...	60	1.40	None
6	50-lb flooring...	50	$1.30 + 0.18 = $1.48	3.5 lb. @ 5 cts = $0.18
7	40-lb flooring...	40	1.40 + 0.36 = 1.76	9 lb. @ 4 cts = 0.36
8	Open-grate flooring......	15	1.40 + 0.48 = 1.88	12 lb. @ 4 cts = 0.48

Steels

The economics involved may depend somewhat upon the character of the steel adopted, because it is conceivable that one type of flooring for a certain span length might be more economical than another type in a carbon-steel bridge, but not in a silicon-steel bridge; hence, curves had to be plotted for both steels. Structures of both materials were investigated for simple-truss bridges, but only silicon-steel structures for cantilever bridges (see Type *A* and Type *C*, Fig. 9), because it is now found to be uneconomical to use carbon steel in cantilever-bridge construction, except sometimes for floor systems and, generally, for certain parts of the lateral bracing. As, in simple-truss bridges, it is not economical to utilize carbon steel for long spans, the carbon-steel curves have been stopped at 400-ft span lengths.

Roadways and Sidewalks

A 45-ft and a 20-ft clear roadway were adopted as the standard floor widths for this investigation, partly because they saved much interpolation on the diagrams of "percentage ratios" that were used in making the calculations for truss weights. This width of 45 ft is practically equivalent, as far as both weight and cost are concerned, to the very common floor cross-section consisting of a 40-ft, clear roadway and two 5-ft, clear sidewalks, for the

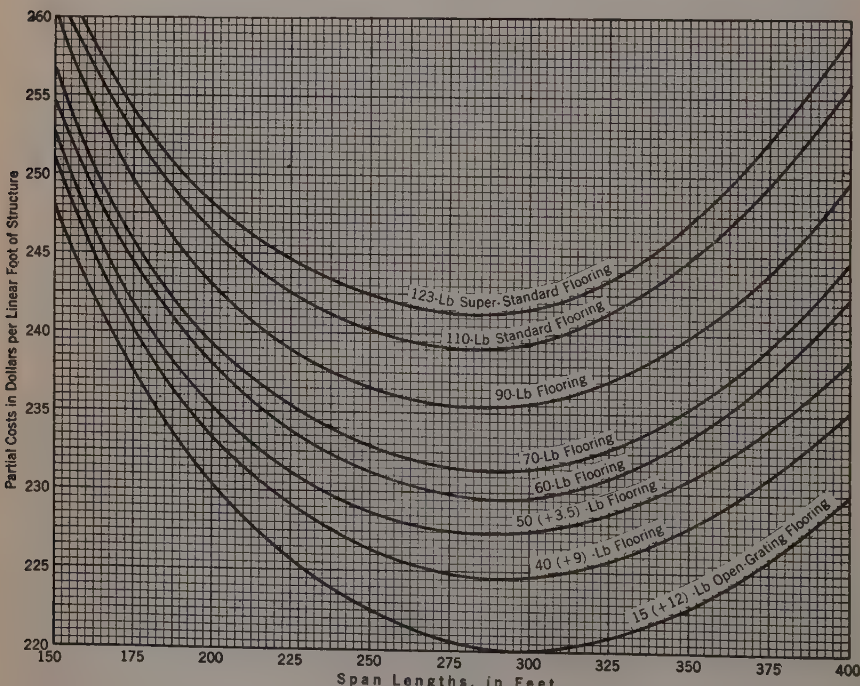


Fig. 6.—Partial Unit Cost of Simple-Truss, Highway Bridges of Silicon Steel; 20-Foot Clear Roadway; Spans, 150 Feet to 400 Feet.

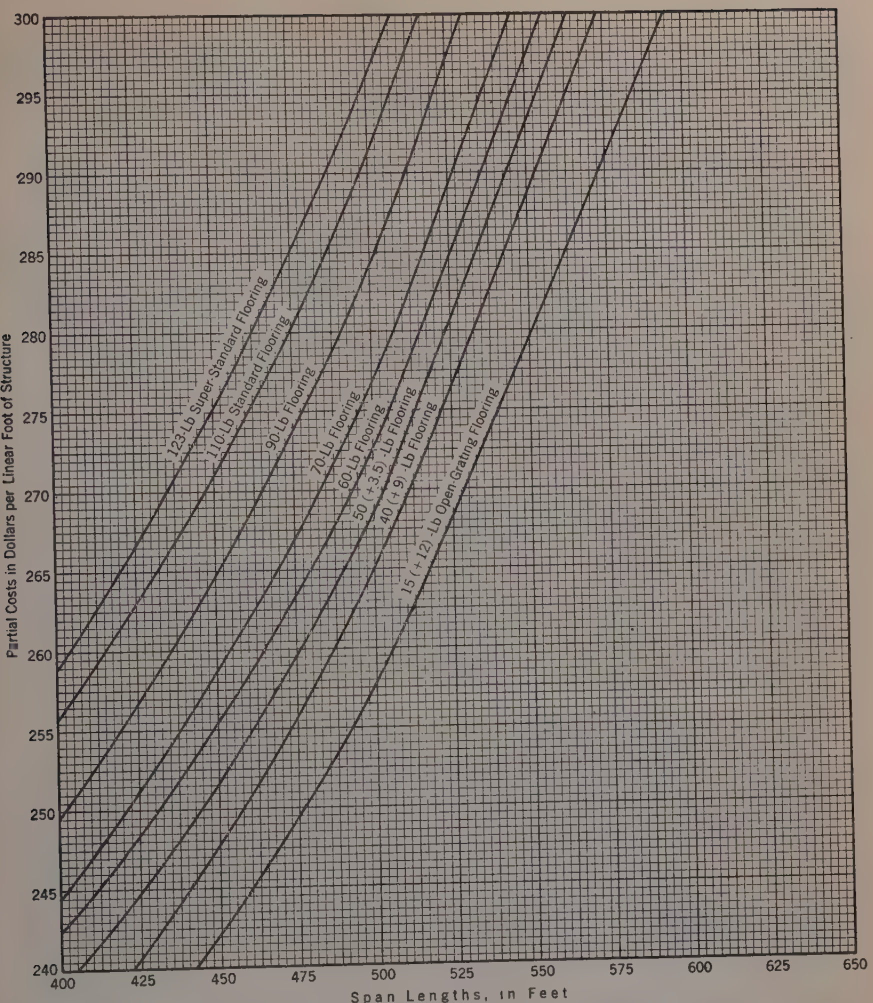


FIG. 7.—PARTIAL UNIT COST OF SIMPLE-TRUSS, HIGHWAY BRIDGES OF SILICON STEEL; 20-FOOT CLEAR ROADWAY; SPANS, 400 FEET TO 650 FEET.

reason that an ordinary reinforced concrete sidewalk weighs and costs about one-half as much per square foot as the ordinary main roadway.

HAND-RAILS

In computing dead loads for the various spans that were estimated, an allowance of 120 lb per lin ft of span was made for the weight of two hand-rails, but their cost has been (and in the future use of this paper is to be) ignored, as it is common to all the competing structures investigated.

GUARD-RAILS

Two types of guard-rails have been adopted for this investigation: (a) 12 by 12-in. curbs of reinforced concrete for structures with solid floors;

and (*b*) rolled channel guards for structures having all-steel flooring, including those of the open-grate type. In Type (*a*), the flooring extends beneath the curbs (or guards) so as to provide an attachment to them sufficiently strong to resist shock from passing vehicles; but, in Type (*b*), the channel

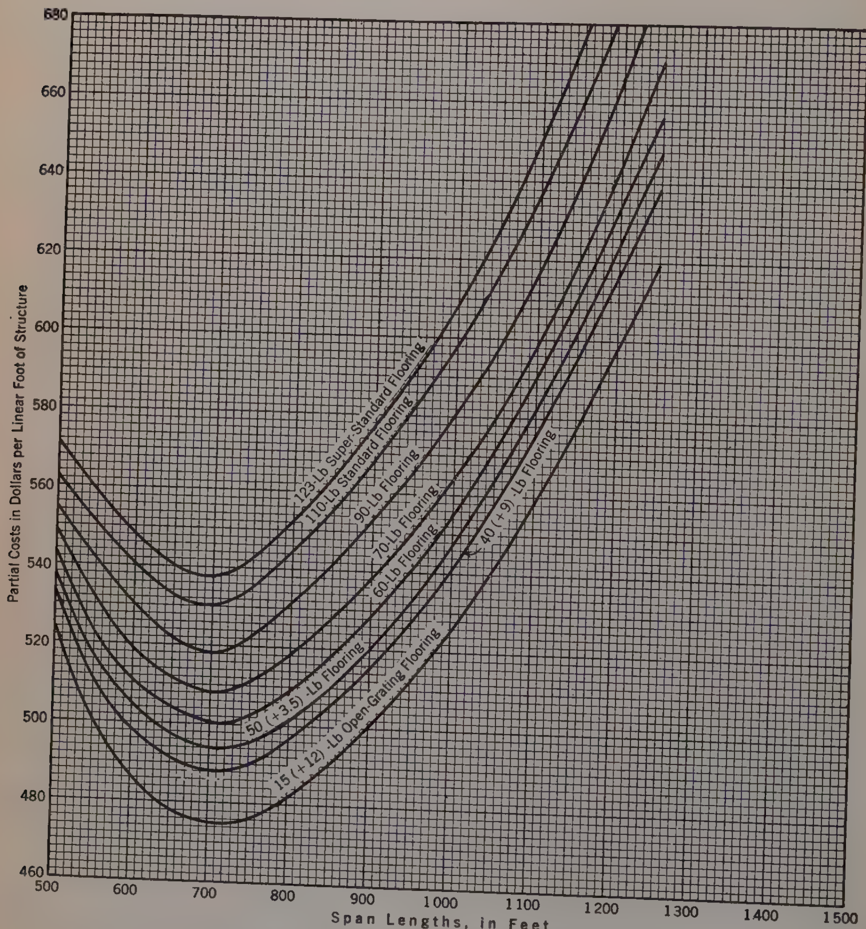


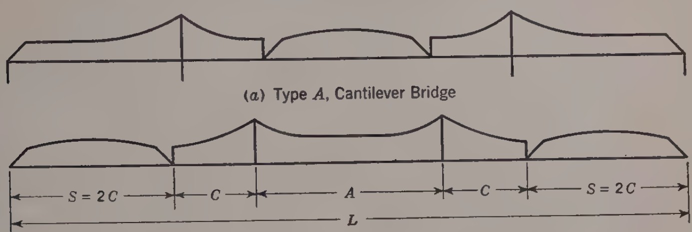
FIG. 8.—PARTIAL UNIT COST OF TYPE *A* CANTILEVER HIGHWAY BRIDGES OF SILICON STEEL; 20-FOOT CLEAR ROADWAY.

guards are to be rigidly attached to the floor system by metal braces. The weight of carbon steel in the two channel guards and their bracing is 100 lb per lin ft, and the cost, in place, is $4 per lin ft.

The cost of a structure for the two reinforced concrete guards and their supporting base is $2 per lin ft for the 2 cu ft of concrete in the curbs, plus the value of 2 sq ft of the flooring. It is self-evident that there is no need for extending the flooring outside the inner faces of the channel guard-rails.

Specifications

The standard specifications that were used for making the computations are those now current, involving basic tensile unit stresses of 18 000 lb per sq in. for carbon steel and 24 000 lb per sq in. for silicon steel.



(a) Type A, Cantilever Bridge

(b) Type C, Cantilever Bridge

FIG. 9.

Although all the data in this paper are based on the usual standard (H-20) live loading, the general economic conclusions will apply quite closely to structures designed for lighter live loadings. For the usual H-15 loading, for instance, the 5-ft stringer spacing could be increased for all types of floor, or it could be kept unchanged and thinner floor-slabs utilized. In general, the percentage saving resulting from the use of the lighter floorings will be found a little less for the lighter live loads than for the H-20 loading.

Dead Load Reduction Effect on Piers

Any reduction in dead load saves money, not only on the superstructure, but usually also on the substructure because it is the sizes of shoe-plates that dictate both the width and the length of the pier tops; hence, it was necessary to include the cost of the substructure in making the comparative curves of costs per linear foot of bridge.

Location Effect

As bridge costs differ somewhat in various parts of the United States, it was considered advisable to select a particular locality for the comparison; and Kansas City, Mo., was chosen as being centrally located.

Additional Metal for Supporting Thin Floorings

Thin floorings may require more closely spaced supports than the "standard" flooring (Item No. 2, Table 1). There are two ways of accomplishing this requirement—either by inserting additional stringers, or by adhering to the 5-ft spacing that is used with the "standard" flooring and crossing the stringers with small I-beams, spaced as far apart as the strength and the rigidity of the flooring under consideration will permit. The second method is the better one from the point of view of stiffness; and it is sometimes also the more economical.

As none of the usual floorings less than 3 in. in thickness will permit of a support-spacing in excess of 3 ft, and as the cross-I-beam construction

is usually so much more satisfactory, the writer concluded to utilize it for such thin floorings. For floorings weighing 60 lb per sq ft, or more, or for those $3\frac{1}{2}$ in., or more, thick, he assumed them to be able to carry properly over a 5-ft spacing. For thin floorings ranging from 3 to $3\frac{1}{2}$ in., he adopted a stringer spacing of 4 ft; and for all still thinner floorings the small transverse I-beams were used, their spacing being dependent upon the strength and rigidity of the floorings. The greatest, safe, usable, span length of any flooring is so difficult to compute that tests to destruction may be needed to determine its strength.

Before using this paper for ascertaining the economics of any particular flooring, the designer will have to satisfy himself as to this safe span length, l (less than 5 ft). He can find, accurately enough for his purpose, the corresponding excess weight of floor metal, e, per square foot of floor by the formula,

$$e = 3.5\ (5 - l) \qquad (1)$$

For instance, if $l = 4$ ft, e will be 3.5 lb.

Trusses

The truss weights were readily obtained by a method presented[2] by the writer in 1935. In Type *A* cantilever bridges[2] they were the properly adjusted average weights for the entire structure, namely, one suspended span, two cantilever arms, and two anchor arms, the metal in anchorages and on piers being omitted

Correction for Effect of Wind Load

It was anticipated originally that the computed truss weights would require correction to cover the effect of wind loading upon the sectional areas of certain bottom-chord members for all the various floorings, except the "standard" type, in which this effect had already been included. Such was found not to be the case for the simple-truss spans with the 45-ft, clear roadway; but it held true for the simple-truss spans with the 20-ft, clear roadway, and for all cantilever spans.

These corrections were determined by reference to several actual bridges designed with "standard' floorings, in which the wind loads had increased some of the bottom-chord sections. The weight of extra metal that would be required to care for the wind loading, if the dead load were reduced by using the lightest of all the floorings, was also computed. Cases of both wide and narrow roadways, involving both carbon steel and silicon steel, were selected; and enough of them were introduced to render this method of correction for wind loading perfectly satisfactory. The corrections for intermediate floorings were interpolated by proportionate weights per square foot of flooring.

[2] "Weights of Metal in Steel Trusses", *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1.

Narrow Roadway

Were it not for the aforementioned feature of correction for the effects of wind loading, the comparative economics of the various types of flooring might have been assumed as constant for all widths of roadway; but, unfortunately, this feature is much more potent for narrow roadways than for wide ones; hence, the writer deemed it advisable to prepare the curves in Figs. 5 to 8, inclusive, in order to cover highway bridges of 20-ft clear roadway. The methods of calculation used were the same as those for the structures of the 45-ft clear roadway, except only that carbon-steel floor systems were adopted in otherwise silicon-steel bridges for Figs. 6 and 7 and carbon-steel stringers with silicon-steel cross-girders for Fig. 8, and that the pound prices for metal erected were duly adjusted for the mixed steels. Proper cognizance was taken of the increased weights of metal in floor systems due to the necessary spreading of trusses in long-span, narrow-roadway bridges, in order to comply with standard engineering practice.

Unit Prices of Steel Erected

The following unit prices of steel, erected, were adopted as representing the average current values at Kansas City, including a proper profit to the contractor, for bridges with the "standard" flooring:

Simple-truss structures of carbon steel........5 cts per lb
Simple-truss structures of silicon steel........6 cts per lb
Cantilever structures of silicon steel..........7 cts per lb

For the mixed steels used in computing the curves of Figs. 3 and 4, the average unit prices for bridges with the "standard" flooring ranged uniformly from 5.4 cts at 150-ft spans, to 5.7 cts at 600-ft spans; and for those used in computing the curves of Fig. 5, they ranged from 6.65 cts at 500-ft main openings to 6.85 cts at 1 200-ft main openings.

For structures with decks lighter than the "standard," a correction in the unit prices of the metal work was made, because, as the weight of metal in a structure is reduced, its pound price, erected, will increase. This correction was effected by means of the formula,

$$C' = \frac{C}{4}\left(3 + \frac{w}{w'}\right) \qquad \text{(2)}$$

in which w = the weight of metal, in pounds per linear foot, in the bridge with the heavier deck; w' = the weight of metal, in pounds per linear foot, in the bridge with the lighter deck; C = the unit cost of metal, erected, in the bridge with the heavier deck; and C' = the unit price of metal, erected, in the bridge with the lighter deck.

Substructures

In determining the costs for substructures, it seemed desirable to make a distinction between the assumed profiles of crossings for simple-truss structures and those for cantilever structures. Generally, the grade line is

lower in the former than in the latter, and the depths below the water-line of the pier foundations are often less. For the simple-truss bridges, the height of the shafts was taken at 42 ft, and the height of the pneumatic bases at 50 ft, whereas, for the cantilever bridges, both the shafts and the bases were assumed to be 84 ft high.

The all-around batter adopted for all piers was $\frac{1}{2}$ in. to the foot. Dumb-bell shafts, as illustrated in plan by Fig. 10, were adopted for all cases, because of their economy in materials and cost. For the simple-truss bridges, with 45-ft roadways, two separate bases were assumed; but for the bridges with 20-ft roadways, and for all cantilever structures, single bases were adopted. The reason for the single bases in narrow structures is that there would probably be no great pecuniary advantage in dividing them; and their use in cantilevers is generally called for because of location over navigable streams where the bed-rock is deep, and where, consequently, the sinking of small bases would be expensive per unit of volume.

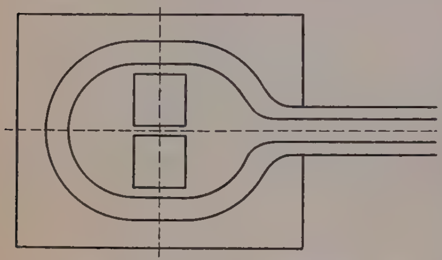


FIG. 10.—PLAN OF PIER TOP.

In computing the costs per linear foot for the substrutures of simple-truss bridges, the total value of one pier was estimated, and the amount was divided by the distance between pier centers; whereas in the case of cantilever bridges, the total value of a main pier was divided by one-half the length of the main opening plus one-half that of one anchor arm. This method assumes that one-half of each anchor arm pertains entirely to its anchor pier. As the design for any anchorage varies greatly with the foundation conditions, it is not advisable to assume the latter, but, instead, to designate the cost per linear foot of the substructure the amount computed from the value of one main pier, as previously described. If this is not strictly exact, no harm can be done, because all the competitive costs per linear foot are affected alike.

UNIT PRICES FOR SUBSTRUCTURE

For the masses of materials in the shafts and walls and in the pneumatic bases of piers, the following unit prices were adopted for structures with the "standard" flooring:

Simple-Truss Structures:

- Shafts and walls ....$15 per cu yd
- Bases ....$22 per cu yd

Cantilever Structures:

- Shafts and walls ....$17 per cu yd
- Bases ....$25 per cu yd

For structures with decks lighter than those of the "standard" flooring, a correction in substructure unit prices was applied, as in the case of super-

structure metal, because, as the size of the pier decreases, the unit prices of shafts, wall, and bases will increase. This correction was made by means of the formula:

$$C' = \frac{C}{4}\left(3 + \frac{V}{V'}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which V = total volume of shafts, wall, and base or bases, in pier with heavier deck; V' = total volume of shafts, wall, and base or bases, in pier with lighter deck; C = one of the substructure unit prices for pier with heavier deck; and, C' = the corresponding substructure unit price for pier with lighter deck.

In respect to the application of the unit-price generic factor,

$$C' = \frac{C}{4}\left(3 + r\right), \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

each of the sets of curves in this paper, was treated as an independent entity; in other words, these factors do not function from diagram to diagram or from span length to span length, but only, in each diagram and for each span length, between the "standard" flooring and the other floorings.

Pier Designing

In pier designing, all steel pedestals were made square; and, in simple-truss structures, 6 in. clear were left between them. In these designs from the edge of any pedestal base-plate to the edge of the masonry, a minimum distance of 9 in. was allowed. The heavier of the two similar piers was first designed and estimated, and then the dimensions were reduced (always exactly and in like manner) to agree with the lighter superimposed load on the pedestal or pedestals. The interpolation of pier costs for the other types of flooring was done with due reference to the superimposed loads and to the varying unit-prices of materials in place.

For convenience, all piers were assumed to rest on solid rock. Had pile foundations been adopted, a still more economic showing would have been indicated for the light floorings. In computing volumes of shafts, the prismoidal-formula method was used invariably; but for the walls, the simpler method of averaging end areas was deemed to be sufficiently accurate. The permissible load on concrete under pedestals was taken at 600 lb per sq in., according to the latest standard specifications of the American Association of State Highway Officials.

Effect of Pile Foundations

It has been stated that, had pile foundations been adopted instead of those of solid rock, a better showing would have been made for the lighter floorings. For example, consider a foundation with 100-ft, large-diameter, timber, friction piles, embedded 10 ft in the concrete. A bearing capacity of 45 tons per pile was assumed, the cost per linear foot of pile in place below the base being \$1.00, and the cost per cubic yard of the mass of base

(placed by a steel sheet-pile coffer-dam) being $20.00. The costs of the two piers thus computed per linear foot of span, for the "standard" and the open-grate floorings, are $98.44 and $82.20, respectively, indicating an advantage of 16.5% in the substructure for the lightest flooring when compared with the "standard." The longer the span the greater would be this percentage of advantage. The corresponding percentage for piers resting on bed-rock is 14.5%, showing that the lightest flooring is only a little, if any, more economical with pile foundations than with rock foundations.

Effect of Excess Weights in Floor Systems

Before proceeding to show how to use the cost curves for actual cases of competition, it is necessary to indicate how to deal with any excess weight of metal per linear foot of span, as compared with the allowance therefor recorded in Table 1. If a competitor has a flooring so thin that it is not strong enough properly to carry over a 5-ft stringer-spacing, it will be necessary, as previously indicated, to put in either additional stringers, or small transverse I-beams, thus adding a number of pounds (n) per linear foot to the total load to be carried. It is then necessary merely to add to the cost of the floor the product of n by the pound price of the metal erected, and to include the extra weight of the metal per square foot of floor when entering the various diagrams

Diagrams of Partial Costs

In the curves accompanying this paper are recorded the computed partial costs (including substructure, trusses, lateral systems, and floor system) per linear foot of structure, for the various types of bridges and the floorings listed in Table 1 As can be seen from these diagrams, the simple-truss spans vary from 150 ft to 600 ft in length, and the Type *A* cantilever spans from 500 ft to 1 200 ft of main opening, all measured from center to center of piers.

On the lowest three curves of each diagram there is noted the weight of extra floor metal resulting from close stringer spacing or small cross-beams, as assumed in Table 1, as well as the weights of the flooring itself, because the trusses and substructure were designed to carry the weight of this extra metal These three curves, therefore, should be considered as representing floors weighing 535 lb, 49 lb, and 27 lb per sq ft.

Type C Cantilevers.—The curves for the Type *A* cantilever bridges can be used for Type *C* cantilever bridges by multiplying the main opening of the latter by 1.43.

Method of Computing the Total Costs of a Structure

References have been made in the text to the fact that the several diagrams record partial costs of bridges per linear foot of structure, and it was indicated that the curves covered only the cost of the substructure and that of the trusses, lateral system, and floor system upon the basis of a 5-ft spacing of floor stringers. In order to find the total cost per linear foot for any case, there must be added to the data given in the diagram the costs

per linear foot for: (*a*) The flooring between curbs; (*b*) any extra stringers, at the unit price, erected, of the metal for the entire bridge; or, (*c*) the small cross-girders that rest on the stringers, worth in place only 4 cts per lb; and, (*d*) the steel guard-rails or the reinforced concrete curbs, as the case may be.

TABLE 2.—ADDITIONAL COSTS OF STRUCTURES, SUPPLEMENTING DATA FROM THE CURVES, FOR FINDING TOTAL UNIT COSTS

Types of flooring	Costs of flooring per linear foot of bridge	Costs per linear foot of extra metal in floor system	Costs per linear foot of curbs or guard-rails	Summation, giving total extra costs per linear foot to add to diagram, readings
(*a*) DATA TO SUPPLEMENT FIGS. 1 TO 4, INCLUSIVE, FOR WIDE HIGHWAY BRIDGES				
Super-standard....	45 ft. @ \$1.00 = \$45.00	None	\$2.00 + \$2.00 = \$4.00	\$49.00
Standard.........	45 ft. @ 80 cts. = \$36.00	None	1.60 + 2.00 = 3.60	39.60
90-lb............	45 ft. @ \$1.00 = \$45.00	None	2.00 + 2.00 = 4.00	49.00
70-lb............	45 ft. @ \$1.50 = \$67.50	None	3.00 + 2.00 = 5.00	72.50
60-lb............	45 ft. @ \$1.40 = \$63.00	None	2.80 + 2.00 = 4.80	67.80
50-lb in carbon-steel bridges....	45 ft. @ \$1.30 = \$58.50	45 ft. @ 18 cts. = \$8.10	2.60 + 2.00 = 4.60	71.20
50-lb in simple-truss bridges of silicon steel	45 ft. @ \$1.30 = \$58.50	45 ft. @ 21 cts. = \$9.45	2.60 + 2.00 = 4.60	72.55
50-lb in cantilever bridges of silicon steel	45 ft. @ \$1.30 = \$58.50	45 ft. @ 24.5 cts. = \$11.03	2.60 + 2.00 = 4.60	74.13
40-lb............	45 ft. @ \$1.40 = \$63.00	45 ft. @ 36 cts. = \$16.20	4.00	83.20
Open-grating.....	45 ft. @ \$1.40 = \$63.00	45 ft. @ 48 cts. = \$21.60	4.00	88.60
(*b*) DATA TO SUPPLEMENT FIGS. 1 TO 4, INCLUSIVE, FOR NARROW HIGHWAY BRIDGES				
Super-standard....	20 ft. @ \$1.00 = \$20.00	None	\$2.00 + \$2.00 = \$4.00	\$24.00
Standard.........	20 ft. @ 80 cts. = \$16.00	None	1.60 + 2.00 = 3.60	19.60
90-lb............	20 ft. @ \$1.00 = \$20.00	None	2.00 + 2.00 = 4.00	24.00
70-lb............	20 ft. @ \$1.50 = \$30.00	None	3.00 + 2.00 = 5.00	35.00
60-lb............	20 ft. @ \$1.40 = \$28.00	None	2.80 + 2.00 = 4.80	32.80
50-lb in carbon-steel bridges....	20 ft. @ \$1.30 = \$26.00	20 ft. @ 18 cts. = \$3.60	2.60 + 2.00 = 4.60	34.20
50-lb in simple-truss bridges of silicon steel	20 ft. @ \$1.30 = \$26.00	20 ft. @ 21 cts. = \$4.20	2.60 + 2.00 = 4.60	34.80
50-lb in cantilever bridges of silicon steel	20 ft. @ \$1.30 = \$26.00	20 ft. @ 24.5 cts. = 4.90	2.60 + 2.00 = 4.60	35.50
40-lb............	20 ft. @ \$1.40 = \$28.00	20 ft. @ 36 cts. = \$7.20	4.00	39.20
Open-grating.....	20 ft. @ \$1.40 = \$28.00	20 ft. @ 48 cts. = \$9.60	4.00	41.60

These additional costs in wide highway bridges for the flooring types and the unit prices given in Table 1 are presented in Table 2(*a*). These data will supplement Figs. 1 to 4, inclusive, for finding the total unit costs of wide highway bridges. The corresponding data for narrow highway bridges, to supplement Figs. 1 to 4, inclusive, are given in Table 2(*b*).

How to Use the Cost Curves

Assume, for example, that an engineer is contemplating the building of a four-lane, Type *A*, cantilever bridge, having a main opening of 750 ft, and two 5-ft sidewalks, and that he intends using a solid flooring. Two types of floor have been shown him, one by Mr. Smith and another by Mr. Jones. Either type would be satisfactory; hence, his choice is merely a matter of economics.

The flooring offered by Mr. Smith is 3 in. thick, weighs 58 lb per sq ft, and costs $1.23 per sq ft in place; but it has a superior limit of only 4 ft between supporting girders, necessitating 3.5 lb of extra metal per square foot of floor, worth 7 cts per lb in place.

The flooring shown by Mr. Jones, is 2½ in. thick, weighs 45 lb per sq ft, and costs $1.18 per sq ft in place; but it requires, for a stringer spacing of 5 ft, small transverse I-beams, which he justly claims will weigh only 8 lb, and will cost only 32 cts per sq ft of floor.

Mr. Smith's Case.—Turning to Fig. 4, and interpolating for 58 + 3.5 = 61.5-lb flooring, the "partial cost" indicated is $680 per lin ft of bridge. The remaining cost per linear foot is found thus:

Flooring, 45 ft @ $1.23	$55.35
Extra metal in floor system, 45 × 3.5 = 158 lb worth 7 cts per lb, or	11.06
Curb supports, 2 × $1.23 = $2.46. Adding $2.00 for curbs makes	4.46
Summation	$70.87
Plus partial cost from Fig. 4	680.00
Total cost of bridge with Mr. Smith's flooring	$750.87

Mr. Jones' Case.—Turning to Fig. 4, and interpolating for 45 + 8 = 53-lb flooring, the "partial cost" indicated is $666 per lin. ft. The remaining cost per linear foot is found thus:

Flooring, 45 ft @ $1.18	$53.10
Extra metal in floor system, 45 × 8 = 360 lb worth 4 cts per lb, or	14.40
Steel guard-rails, 100 lb @ 4 cts	4.40
Summation	$71.50
Plus partial cost from Fig. 4	666.00
Total cost of bridge with Mr. Jones' flooring	0737.50
Difference in favor of Mr. Jones = $750.87 − $737.50 =	$13.37

To assume another case—a succession of wide, simple-truss, carbon-steel spans of 250 ft each: Mr. Brown thinks he can compete with the standard

flooring by quoting a price of \$1.30 per sq ft of floor for his type of flooring, which is 3 in. thick and weighs 50 lb per sq ft. It requires a 4-ft spacing of stringers, necessitating 3.5 lb per sq ft of extra metal for stringers, worth 5 cts per lb in place—the same as the 50-lb flooring in Table 1.

Turning to Fig. 1, the "partial cost" for 50 + 3.5 = 53.5-lb flooring is found to be \$343.50. The remaining cost per linear foot is found thus:

Flooring, 45 ft @ \$1.30	\$58.50
Extra metal, 45 × 3.5 = 158 lb @ 5 cts	7.90
Curb supports, 2 × 1.30 = \$2.60. Adding \$2.00 for curbs makes	4.60
Summation	\$71.00
Plus partial cost from Fig. 1	343.50
Total cost with Mr. Brown's flooring	\$414.50
For the "standard" flooring, Fig. 1 indicates a partial cost of	\$373.20
Table 2(a) gives a remaining cost as	39.60
Plus the total cost with "standard" flooring is	412.80

Mr. Brown, consequently, does not secure the contract.

The following is an illustration of how to apply the diagrams to a comparison of the economics of an open-grate flooring and the "standard" flooring: An engineer has a 4-lane, simple-truss, silicon-steel, highway bridge, with a succession of 380-ft spans and double, 5-ft sidewalks, to design and build, and desires to know what percentage of saving in cost can be obtained by means of an open-grate flooring which has the longitudinal pieces, $\frac{1}{4}$ in. thick. The best quotation he can secure for the flooring is \$1.50 per sq ft in place. The open-grate flooring, of course, will be the criterion to use; but, as previously indicated, the thickness of the longitudinal pieces adopted for this type in Table 1 was $\frac{3}{16}$ in. The increased thickness adds 2 lb per sq ft of floor to the dead load, but reduces the weight of the special cross-girders from 12 lb to 11 lb per sq. ft, thus making the net increase in total load, 1 lb per sq ft, or 45 lb per lin. ft of span. Turning to Fig. 2 the "partial costs" in pounds per linear foot, is found to be as follows:

Standard flooring	\$411.00
Open-grate flooring (for 17 + 11 = 28 lb)	353.60
Table 2(a) gives for the remaining cost of the standard flooring	39.60
Making the total cost for the bridge with standard flooring, \$411.00 + \$39.60	450.60

For the open-grate structure the additional cost is found thus:

Flooring, 45 × 1.50	$67.50
Extra metal in floor system, 11 × 45 = 495 lb @ 4 cts	19.80
Guard-rails	4.00
Summation	$91.30
Plus partial cost from Fig. 2	353.60
Total	$444.90

The ratio of costs, therefore, will be $\frac{\$444.9}{\$450.6} = \$0.987$. Hence, the saving will be 1.3 per cent.

Movable Spans

This investigation has thus far ignored an important economic application of light floorings, namely, their use in vertical-lift and bascule bridges. In these cases, regardless of span length, there is a decided saving of money in adopting any of the light floorings, especially an open-grate flooring, because the economy involved is not confined (as in fixed spans) to the floor system and the trusses, but extends to the cables with their connections, the counterweights, the sheaves with their bearings, the towers, the entire operating machinery in vertical-lift bridges and bascules, and to numerous other parts of bascule bridges—all in addition to the saving in cost of substructure.

In the past, almost all bascule spans have been designed with light flooring, first of timber (which is most objectionable because of fire risk) and, later, of some patented type of thin flooring; but many vertical-lift spans have had ordinary flooring of reinforced concrete; and the weight of this flooring constitutes a considerable portion of the total load to be raised and lowered.

The open-grate flooring has an additional advantage for bascule spans, shared by no other type of flooring: When the span is being raised or lowered, the force of the wind blowing through the grating largely reduces the pressure against the floor, thus lessening materially, under certain extreme conditions of operation, the quantity of power required to raise or to lower the moving leaf.

The open-grate flooring has a characteristic that is of considerable economic importance in vertical-lift bridges—it does not have a tendency to collect snow or ice. Therefore, the customary excess-load allowance for snow or ice of 5 lb per sq ft of floor in such structures may safely be reduced, thus effecting quite a saving in total cost of structure and of the operating machinery, in addition, of course, to the numerous economies from the reduced weight of the flooring itself.

The combination of all these savings by the open-grate flooring on both vertical-lift and bascule spans is so large, in comparison with the ordinary reinforced concrete deck, as always greatly to overcome the handicap of the

excess unit-price of the lighter flooring. At present (1937) it is not worth while to attempt to investigate the comparative economics of the various light floorings on movable spans; but at some time in the future it might prove otherwise.

Trestle Bridges

In view of the present, very intensive introduction of elevated highway structures to relieve the congestion of traffic in metropolitan centers, the writer has been urged to treat the economy of floors on such structures.

In general, such structures are composed essentially of short spans, and are similar in layout to elevated railways in cities and to ordinary trestles. In such cases there is absolutely no economic advantage in lightening the floor by increasing, materially, its cost per square foot. The usual thick, reinforced concrete flooring. therefore, is the best type to adopt for both economy and rigidity, providing the matter of lateral skidding is ignored; but should such structures contain spans exceeding 250 ft in length, a lighter flooring might be economical on these longer spans.

Suspension Bridges

The economics of the various types of flooring for suspension bridges may be stated in a very few words—the lighter the flooring the less will be the first cost of the structure, and, therefore, the open-grate type would seem to be called for in all cases; but sometimes the designer may desire to rely upon continuity in the flooring for lateral rigidity, because the usual narrow suspension bridge is unquestionably lacking in that desideratum. In such a case, it might be well to use a thin, solid flooring; but the writer's preference would be to adopt the open grate and provide a rigid, substantial lateral system near the plane of the deck.

A study of Figs. 4 and 8, in conjunction with Table 2, should convince any one that heavy floorings are always uneconomical in cantilever bridges, and that the open-grate flooring is specially economical for this type. It is conceded that a thick, solid flooring on the anchor arms would reduce the uplifts at the anchorages, and thus effect a saving in the truss weights of cantilever bridges, as well as in the cost of the flooring itself; but such a saving would not be as great as the gain from the reduced dead loads on the entire main span caused by the open-grate, which reduction affects favorably all the trusses in the suspended span, the cantilever arms, and the anchor arms. It is true that the open grating could be used on the main span, and the heavy flooring on the anchor arms; but the hybrid deck thus produced would be objectionable from the æsthetic viewpoint, and, possibly, also for other reasons.

Warnings

Some one may claim that a fundamental change in the governing unit prices for materials and labor in connection with bridge building will render null and void the service of the cost-curve diagrams; but such is by no means the case, because the relativity of the results will still hold good, and

the comparisons of economics will remain reliable, in spite of any such changes. It is true that the unit prices for superstructure and those for substructure do not always vary in the same proportion; but eventually they settle down to an almost constant relationship. Even a permanent dislocation of their relativity, however, would not cause any material incorrectness in the results of the application of the curves to their legitimate function.

Conclusion

Attention is called to the fact that no where in this paper has any reference been made concerning the superiority, or the contrary, of any particular patented flooring; the problem investigated is solely one of economics. A consistent effort has been made to avoid advocating "special interests," with the understanding that discussers, likewise, will confine their comments to the intended scope of the paper.

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DISCUSSIONS

ADMINISTRATIVE CONTROL OF UNDERGROUND WATER: PHYSICAL AND LEGAL ASPECTS

Discussion

BY WELLS A. HUTCHINS, ESQ.

WELLS A. HUTCHINS,[43] ESQ. (by letter).[43a]—A scholarly discussion of ground-water law in the United States and of the physical situations to which it applies, is presented in this paper. Mr. Conkling's conclusions will merit study by those concerned with legislation pertaining to underground waters.

Status of Ground-Water Law in Several States.—Under the heading, "Underground Water Law in General", a statement is made as to the status of the law governing such waters in each of the seventeen Western States. Additional comments in the case of several States may be in point.

Arizona.—The water code provides that "the water of all sources, flowing * * * in definite underground channels" belongs to the public and is subject to appropriation. The Courts have held, as Mr. Conkling states, that waters percolating generally through the soil belong to the land-owner and that the rule of reasonable use applies, subterranean streams flowing in natural channels between well-defined banks being subject to appropriation under the same rule as surface streams.

The decision in Maricopa County Municipal Water Conservation District Number One *et al. v.* Southwest Cotton Company *et al.* (39 Ariz. 65, 4 Pac. (2d) 369), held that the presumption is that underground waters are percolating in their nature, and that one who asserts that they are not must prove his assertion by clear and convincing evidence; furthermore, that before an underground stream becomes subject to appropriation, there must be certainty

NOTE.—The paper by Harold Conkling, M. Am. Soc. C. E., was published in April, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1936, by Messrs. Joseph Jacobs, and W. D. Faucette and J. E. Willoughby; September, 1936, by R. E. Savage, Assoc. M. Am. Soc. C. E.; December, 1936, by Messrs. H. J. F. Gourley and O. J. Baldwin; and January, 1937, by Messrs. G. E. P. Smith, and David G. Thompson. Approved for publication in *Proceedings*, Am. Soc. C. E., by the Bureau of Agri. Eng., U. S. Dept. of Agriculture.

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[43a] Received by the Secretary December 16, 1936.

of location as well as of existence of the stream. Whether surface or subterranean, a watercourse is held to be essentially a channel, consisting of a well-defined bed and banks, and a current of water which need not flow continuously. Hence, a prior appropriator from a surface stream, who asserts that his rights are being infringed by pumping plants in a valley traversed by the stream, must demonstrate this physical interference, according to the foregoing rule, "by clear and convincing evidence."

Nevada.—Important additions and amendments to the ground-water law were made in 1935. The State Engineer is to designate underground areas and sub-areas for administrative purposes. No administrative areas, however, have yet been designated.

Utah.—After a line of decisions supporting the correlative doctrine, the Supreme Court of Utah has now (1936) announced the application of the appropriation doctrine to at least some underground waters. The two cases in point are Wrathall *v.* Johnson *et al.* (86 Utah 50, 40 Pac. (2d) 755, January 2, 1935), and Justeson *v.* Olsen *et al.* (86 Utah, 158, 40 Pac. (2d) 802, January 10, 1935).

The Wrathall case went up on demurrer, and the Supreme Court held that the complaint stated a cause of action and that the demurrer should have been over-ruled; but the Justices were not in agreement as to why it should have been over-ruled. Two Justices stated that the doctrine of appropriation and beneficial use should be applied to underground percolating waters, and a third that a cause of action was stated under either doctrine, and that waters in an artesian basin should be distinguished from mere "percolating" or "diffused" waters in privately owned land, not connected with other waters. The other two Justices indicated that the complaint stated a cause of action under only the correlative doctrine.

A week later came the decision in the Justeson case, involving a conflict between owners of adjoining tracts, part of each being underlaid by a common artesian basin. Three Justices applied the appropriative doctrine to an "artesian basin", which is "nothing more than a body of water more or less compact, moving through the soils with more or less resistance." The concurring opinion distinguished such waters from "mere percolating or diffused waters." The two Justices who in the Wrathall case concurred in the results but in favor of the correlative doctrine, dissented in the Justeson case.

Shortly afterward the Legislature declared all waters in the State to be the property of the public, subject to existing rights, and provided complete machinery for appropriating underground waters. To the present time (November 2, 1936) the Supreme Court has not had occasion to construe these provisions. In the meantime, the State Engineer is actively administering the statute, twenty-six underground water areas having been defined to September 28, 1936.

In view of the facts: (*a*) That the recent decisions—all handed down prior to the comprehensive legislation—were rendered by a divided Court, with sharp and definite dissenting opinions; (*b*) that the prevailing opinions classified underground waters; and (*c*) that the legislation covering all under-

ground waters has not yet been before the Supreme Court, it appears that further decisions should be awaited before stating, too conclusively, the law of Utah in regard to underground waters.

Wyoming.—So far as the writer is aware, the only decision of the Wyoming Supreme Court on underground waters is Hunt *v.* City of Laramie (26 Wyo. 160, 181 Pac. 137, June 2, 1919). This holds that, if developed artificially, percolating waters belong to the owner of the land on which they are developed.

Co-Ordination of Surface and Ground-Water Rights.—In discussing the administration of surface streams, the author states that only recently has the connection between surface water and underground water been recognized legally, and that the two are still, to a great extent, treated in separate and distinct doctrines of law. Doubtless, this situation has resulted from the fact that surface water is out in the open for all to see, its courses and boundaries readily ascertainable, and its quantity subject to reasonably accurate measurement; whereas, exactly the reverse is characteristic of underground water, although the technique of estimating ground-water supplies has been greatly developed and improved in recent years.

One of the cardinal principles of the doctrine of prior appropriation is that an appropriative right is entitled to protection on all sources of supply of the stream to which the right attaches. An attempted diversion of water from a tributary, at a time it is needed by prior appropriators on the main stream, will be enjoined; for it was recognized in early decisions that the unlimited acquisition of rights on tributaries would eventually deprive the main-stream prior appropriators of their water supply; and, yet, some of these States which follow the appropriation doctrine exclusively as to surface streams have applied the correlative doctrine to underground waters generally, regardless of the fact that such waters may be an important source of supply of surface streams to which prior rights have attached. Doubtless, these inconsistencies have resulted, in part, from the location of tracts over which the first controversies arose; from the failure of river appropriators to recognize the implications of a suit between owners of land overlying a basin constituting one of their sources of supply, and to intervene to protect their rights; and from the difficulties of proving the physical inter-connections of subterranean and surface waters.

Obviously, underground and surface waters cannot be entirely dissociated. The connection between surface streams and tributary or supporting underground waters is too marked. An aggregate flow of 100 cu ft per sec entering a river is no less tributary, physically, when it rises in numerous places through the bottom or seeps through the sides of the channel than when it spills over the bank. To intercept this flow of 100 cu ft per sec by pumping plants, if underground, or by dams, if a surface flow, reduces the river to the same extent; yet in a correlative-doctrine State, such as California, these methods of interception are subject to different rules of law. In States in which the doctrine of appropriation of all waters is carried to its logical conclusion, such as in Colorado, neither pumping nor surface diversions would be

permitted to interfere with the passage of this 100 cu ft per sec while it is needed to satisfy earlier river priorities. The pumping diversion, however, would have its own priority to the same extent as would a surface interception if it antedated the river appropriation or became vested with a prescriptive right. All priorities, in other words, relate to the stream system—the river and all contributing sources. To divide the surface waters in an area embracing any part of that system according to one set of priorities, and the underground waters according to a different set, without regard to possible inter-relationships, would (if the development proceeded far enough), inevitably infringe sooner or later upon prior river rights and result in a hopeless confusion of water titles.

Co-ordination of rights to surface and underground waters, therefore, appears to be a condition precedent to the most complete utilization of water in areas in which physical interconnections exist. Under the doctrine of appropriation this will involve in a given region the separation of percolating waters tributary to surface streams from those not so tributary; adjudications of existing rights; allocation of the remainder of the water supply to further appropriations at diversion points consistent with those of established priorities; and extinguishment of some existing priorities, either through voluntary agreement or condemnation by public authority, in order that ground-water development may proceed, where the proposed development is sufficiently valuable to justify the expense.

Desirability of Administrative Control.—In summing up the results of his examination of State laws affecting the use of underground waters, the author lists four doctrines of law, three of which recognize ownership of water by the land owner and one by the State. Administrative control will be concerned with this fourth doctrine, under which the title to the ground-water is in the public or the State, subject to beneficial use by individual appropriators. There is a possible field for State control under the correlative doctrine likewise, in basins subject to overdrafts; but if experience with riparian rights on surface streams may be taken as a guide, adjustments of controversies over water in individual ownership will be left to the Courts

When a State is once committed to a doctrine of private ownership of water, change to a public ownership system is difficult. The change is much more feasible before great development has been made, than afterward; before vested rights and their protection offer serious resistance. The Nevada Courts for a period of thirteen years recognized the riparian doctrine as to surface streams, to a certain extent at least, and then abrogated it. Riparian rights embodied in Court decrees became vested, but the abrogation was sufficiently early in the State's development to avoid widespread complications. After a long line of decisions supporting the correlative doctrine of ground-waters, the Utah Court now favors the appropriation doctrine; but only a very small percentage of the irrigated area receives its supply from wells. The situation in California is very different; in that State nearly one-third of the total irrigated area secures water exclusively from wells. A movement now under way to effect a modification of the correlative doctrine in California is faced with difficult constitutional questions.

Undoubtedly, in various areas, development of ground-water exists that would not have existed had control been in effect. Nevertheless, a certain amount of this has taken place at the expense and to the injury of early appropriators. This may or may not be in the public interest in a given locality, depending upon local conditions. In parts of California—a correlative-doctrine State—early users of underground water have been forced to go deeper and deeper into the ground for their water supplies as a result of increasing later use by other owners of land overlying the same basin. Development has been great; but against this must be cited the great increase in cost of operation forced upon the early users, the failures on the part of those who could not stand the mounting costs, and the uncertainty as to available water supply if and when all those who are entitled to pump the water choose to exercise their rights. Under the correlative doctrine, as compared with the appropriation doctrine, those who install pumping plants have little protection in the maintenance of their water right; no matter how long they may use the water, its adequacy and availability may be impaired at any time by the installation of pumping plants by other correlative owners. In other words, a correlative right to pump in an area in which the tillable land exceeds the water supply simply means that when all exercise their rights no one will have enough; if some develop a supply adequate for their needs in the meantime, they will be deprived of at least part of it later.

Extreme application of the appropriation doctrine, on the other hand, is open sometimes to serious objections. For example, the first appropriator is entitled to have the water flow in the stream to his point of diversion when it will do so in quantities sufficient to be useful; and he is not to be deprived of this right in favor of a later appropriator higher up the stream on the ground that the water would be greater in quantity and, consequently, more useful up stream. Regardless of heavy losses in the stream bed, the earlier appropriator is entitled to the flow to the extent of his appropriation. To require 100 cu ft per sec to be released up stream to supply 5 cu ft per sec to the early priority down stream may appear unreasonable, but it is the latter's right. Analogous to this situation would be the prevention of any later development of underground water that would lower the water level at the earliest user's well below the point from which he can afford to pump. The earliest user in each case would have protection under the strict application of the appropriation doctrine. This is a property right, important to the individual. The application in such case is objectionable from the standpoint of the public interest, for it requires an allotment of natural resources to one individual that is capable of serving many to greater advantage.

The appropriation doctrine has proved better suited to the Western States than the riparian doctrine. Nevertheless, the last word in its development has not necessarily been spoken. The author indicates that there is a tendency toward modification; undoubtedly, there is room for modification to include economic use, and for laying more stress upon the element of public benefit.

Granted that the appropriation doctrine as to surface waters, in substantially its present form, will be retained, and that interest in bringing ground-

waters under State control on an appropriative basis has grown in recent years, the key to the future development of ground-waters tributary to a stream system in a strict appropriation State may lie in some form of public or co-operative acquisition and extinguishment of prior rights which interfere with development, compensation to be made to the holders of the rights thus extinguished. The same reasoning applies to extinguishing prior ground-water rights along the rim of a basin, which if not extinguished may prevent the use of a much greater water supply drawn from storage in the interior of the basin. The prospective ground-water use must justify itself however; and if the user can not afford to pay just compensation, the necessity for any such exchange of water right is questionable unless some other public benefit is involved. In the case of surface-water appropriations, the working out of an equitable basis for an adjustment should not present great difficulties. In the case of ground-water it is more complicated, of course, both physically and economically.

Difficulties of Administration.—The author indicates some of the difficulties that face the administrator charged with control of ground-water. It is a complicated proceeding, of course, and it is appreciated that precise determinations may be difficult or impossible in a given case, or so costly as to be prohibitive. Nevertheless, in view of the increasing knowledge and mastery of ground-water hydrology, there seems little reason to doubt that a careful, scientific investigation will yield substantial justice to the elements.

Administration of underground waters will be more expensive than that of surface streams, due to the cost of necessary investigations. That, however, is an element to be considered in any proposed development of ground-waters, just as is the high cost of pumping from underground, as compared with the cost of gravity diversions from surface, streams.

The fact that the administrator must enter the field of economics, from which he has been free in the administration of surface streams, may be an unwelcome departure to the individual, but is not an insuperable obstacle. Methods have been developed for determining costs and returns in farming operations, costs of water, and capacity of lands to pay for irrigation development. The administrator will adapt existing methods in reaching his decisions as to issuance of permits and regulation of pumping drafts, rather than develop an entirely new technique.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

BACK-WATER AND DROP-DOWN CURVES FOR UNIFORM CHANNELS

Discussion

By Messrs. Chesley J. Posey, and A. A. Kalinske

Chesley J. Posey,[9] Jun. Am. Soc. C. E. (by letter).[9a]—A unique method of solving the general equation of motion for steady non-uniform flow in uniform channels is presented in this paper. It is the only method yet published that takes into account the effects of changing velocity head, friction, and channel shape, without any necessity for dividing the reach into steps. The velocity head is assumed to be equal to a constant, α, times the theoretical velocity head based on the average velocity in the cross-section. At present, there is no basis for any other assumption. Consideration of the friction factor is based on an exponential type of formula; no more satisfactory formula is available. The assumption is made that the friction loss for a stream flowing at Velocity V, and Depth D, not the neutral depth, is the same as if there were neutral flow at that velocity and depth. This assumption is open to question, because of the inherent differences between convergent, parallel, and divergent flow. In uniform channels, however, the angle of divergence is not likely to be great enough for serious errors to arise from this source. Consideration of the effect of the shape of the channel is made possible by expressing the area and wetted perimeter as monomial exponential functions of the depth. Simple logarithmic plotting will demonstrate the high degree of approximation that is possible, over a wide variety of channel shapes.

Proof of the accuracy of the author's method is evidenced by the remarkable agreement obtained in the laboratory tests, reported in the paper. These tests covered as wide a range of the variables in question as could be included in a laboratory investigation without unreasonable cost. Data secured from tests in larger and longer channels would be desirable to establish a further check. Additional information might be secured as a by-product of some other investigation, and it is to be hoped that experimental workers and practicing

Note—The paper by Nagaho Mononobe, M. Am. Soc. C. E., was published in May, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1936, by J. C. Stevens, M. Am. Soc. C. E.

[9] Asst. Prof. of Mechanics and Hydraulics, State Univ. of Iowa, and Assoc. Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

[9a] Received by the Secretary December 17, 1936.

engineers will take advantage of opportunities to secure and report further data on back-water and drop-down curves.

The author's method possesses definite advantages over those methods that require the use of steps. A vital part of the curve may be determined quickly without the tedious computation of steps intervening between it and the control point. Furthermore, theoretical curves such as those presented by the author may be used in the study of the shapes of the various possible surface curves, and in determining under what circumstances they will occur and from what feature their position along the length of the channel is fixed. The author applies his method to the two most common types of back-water curves. A complete discussion of the twelve possible types has been given[10] by Sherman M. Woodward, M. Am. Soc. C. E.

Integration, taking into account the shape of the channel, is made possible by the author's empirical Equations (13). Before giving these formulas the author sets up Differential Equation (8), based on Equations (5) in which the hydraulic area, the wetted perimeter, the hydraulic radius, and the average velocity "are expressed most generally as functions of D, the depth of water measured from the lowest line of the channel bed." As can be seen from Equations (24), (28), (29), and (31), Equations (5) are not the most general; as approximations it is doubtful whether they are much better than Equations (13). Since neither Equation (8) nor Equation (12) is integrated, it is difficult to see why Cases 1 and 2 are included in the paper. Their presence, for which there is only a slight logical justification, makes the paper more formidable than it need be.

To determine the values of s and k, the area and wetted perimeter may be plotted logarithmically as functions of the depth, and straight lines approximating the relations over the range desired, drawn in by eye. The values of s and k are determined by the slopes of these lines. This graphical method has the advantage of enabling one to judge the closeness of fit obtained. In cases where there is no question but that a high degree of approximation will be secured, Figs. 9, 11, and 12 may be used.

The writer has been unable to check certain values from these diagrams and believes they may contain errors. For the example of the rectangular section for which the author gives $s = 1.0$ and $k = 0.38$, the writer obtains $s = 1.0$ and $k = 0.41$. For the example of the circular section, for which the author gives $s = 1.34$ and $k = 0.59$, the writer obtains $s = 1.29$ and $k = 0.65$. These results were obtained by the graphical method and can be checked by any one familiar with logarithmic plotting; for this reason it is considered unnecessary to reproduce the graphical work herein. It should be noted, furthermore, that, in Equation (18), $\phi_2 = \int \frac{y^{(r-2s)-1}}{y^r - 1}\, dy$; and, in Equation (30), an equality sign precedes y^s.

The author's method is simple and straightforward. It may justly be considered the culmination of the work started by Belanger and Bresse. In making the method useful to practicing engineers, it will be necessary to

[10] Technical Repts., Miami Conservancy Dist., Pt. III.

simplify the charts of Fig. 3, re-drawing them to a larger scale, and separating the curves for ϕ_1 and ϕ_2, as has been done with ψ_1 and ψ_2 in Figs. 5, 6, and 7.

A. A. KALINSKE,[11] ESQ. (by letter).[11a]—Undoubtedly, the most important contribution by the author is the presentation of data from certain carefully conducted experiments on back-water and drop-down curves and his discussion of the different varied-flow expressions developed by others. Although he does not go into any great detail as to experimental procedure, which is always of interest to those engaged in experimental hydraulics, his general description indicates considerable painstaking experimental and analytical effort. His "Concluding Remarks" should be of particular interest to those concerned with errors introduced by making various assumptions when setting up or using varied-flow formulas.

In developing his expression for back-water and drop-down surface curves the author demonstrates effectively that the most complicating item in the development of any varied-flow expression is the difficulty of obtaining a simple relation between the height of water above the channel bottom, the cross-sectional area of water flowing, and the hydraulic radius. To express A and R in terms of depth always leads to extremely complicated notations and mathematical expressions. Nearly all varied-flow expressions developed have led to such integrations that special tables and curves must be used if a problem is to be solved in any reasonable time. Furthermore, the special notations used require considerable time to master, and, frequently, the fundamental hydraulic principles are obscured. This makes it particularly difficult to present the subject of varied flow to students of hydraulics.

The author's back-water and drop-down functions are quite involved and probably will not be of great use to practicing engineers, or of great value in presenting the general subject to students. In trying to apply the various functions and also other existing varied-flow formulas to non-uniform flow in circular sewers, the writer encountered considerable difficulty, and lost much time in obtaining answers to specific problems.

The author's formulas for total length of back-water and drop-down curves (Equations (18) and (20), respectively) can be used for any length of reach as long as the relations between depth, hydraulic radius, and area remain constant and the channel slope does not vary. However, in many types of cross-sections the constants in the type of monomial used by the author, in expressing the relation between area, hydraulic radius, and depth, vary enough so that good accuracy in outlining the surface curve can only be obtained by solving for distances between successive depths. Furthermore, if a number of points are desired on the surface curve, computation of its total length by solving for the distance between successive depths does not complicate matters, especially if the reaches need not be too short in order to obtain good precision.

[11] Instructor in Hydr. and San. Eng., State Univ. of Iowa, Iowa City, Iowa.

[11a] Received by the Secretary, January 7, 1937.

A considerable number of mathematical operations and notations can be omitted without the sacrifice of any precision or lengthening of time of computation by dispensing with an expression for hydraulic radius in terms of depth (Equation (13c)). Since the hydraulic radius, relatively, changes the least from depth to depth of any of the variables, it is a convenient term to average in any reach; and in the development of the varied-flow integral from the fundamental Bernouilli equality (Equation (3)), worth while simplifications result if the velocity is the variable instead of the depth. Note that the term, $\frac{d\,(V)^2}{2\,g\,dx}$, in Equation (3), is equal to $\frac{V\,dV}{g\,dx}$. For many common types of cross-sections, whether regular or irregular, for a particular range of depths, the area and depth can be conveniently expressed in a straight-line relationship of the type, $D = a\,A + b$. Substituting $\left(\frac{Q}{V}\right)$ for A, the value of $\frac{dD}{dx}$ in Equation (3) becomes $-\frac{a\,Q\,dV}{V^2\,dx}$. The straight-line relationship is much more simple to use since the slope of the line, a, can be so easily obtained; for any given reach, it is equal to $\frac{D_1 - D_2}{A_1 - A_2}$.

Making the foregoing changes and substitutions in Equation (3), the following integral results:

$$dx = \frac{\alpha\,V\,dV}{\frac{g\,V^2}{C^2\,R^{2m}} - S_0\,g} - \frac{a\,Q\,dV}{V^2\left(\frac{V^2}{C^2\,R^{2m}} - S_0\right)} \quad\ldots\ldots\ldots\ldots(37)$$

which, on integration, gives an expression for the distance between any two depths (or velocities):

$$L = \frac{\alpha\,C^2\,R^{2m}}{2\,g}\log_e\left(\frac{V^2_1 - V^2_u}{V^2_2 - V^2_u}\right) - \frac{a\,Q}{S_0}\left(\frac{1}{V_1} - \frac{2}{V_2}\right) - \frac{a\,Q}{2\,S_0\,V_u}\log_e\frac{\frac{V_1 - V_u}{V_1 + V_u}}{\frac{V_2 - V_u}{V_2 + V_u}} \quad\ldots(38)$$

In Equation (38), $V_u = C\,R^m\,S_0^{0.5}$, in which R is the average value of the hydraulic radius between the depths chosen. If the surface curve in the reach approaches a straight line, R is an arithmetical average; if it is considerably concave or convex, the value of R can be slightly decreased or increased to make it approach more truly the average in the section. Using only a slide-rule and arithmetical averages of R, computations were performed on the experimental back-water and drop-down curve data given by the author (see Table 10). Equation (38) applies to either back-water or drop-down curves on sustaining slopes; V_1 is the up-stream velocity and V_2, the down-stream velocity.

The values for C (Table 10) were computed from data given by the author; the value of m in the back-water problem was 0.65 and in the drop-

down problem, 0.70. Note that the increments used are relatively large, and the values obtained are comparable in accuracy to those of the author. Although the expression itself appears quite formidable, the computations are not tedious and can be performed very quickly on a slide-rule. It is also of advantage to be able to use the same expression for either back-water or drop-down curves instead of different functions as is done by the author.

TABLE 10.—COMPUTATIONS PERTAINING TO A SMOOTH RECTANGULAR CHANNEL

D-inches	*V*	*R*	*C*	Increment in *L*	Values of *L*		
					Computed	Test	Mononobe
(*a*) BACK-WATER CURVES; $S_o = 0.002$							
3.937	0.805	0.164	152	0	....	....	....
2.953	1.074	0.140	152	47.4	47.4	47.6	47.8
2.165	1.453	0.116	152	49.5	96.9	98.4	97.0
(*b*) DROP-DOWN CURVES; $S_o = 0.001$							
2.461	2.200	0.126	155	0	....	....	....
3.030	1.790	0.143	155	13.5	13.5	12.4	128.
3.752	1.446	0.160	155	99.0	112.5	101.0	110.0

For a rectangular section, the value of *a*, of course, remains constant; also, for wide trapezoidal sections, it does not change greatly in value for different ranges of depth. For circular sections, *a* does not change much in its value in a range of depths between 10% to 90% of the diameter. For triangular and narrow trapezoidal sections the more accurate relation between depth and area is of the type, $D = k\ A^{0.5}$.

Use of this parabolic relationship complicates the second term of the integral in Equation (37); however, direct integration is quite possible, and is not beyond the scope of the mathematical equipment of the average engineer. The straight-line and parabolic reationship will take care of all types of channels, and there seems no real necessity to complicate matters by using monomials of the type introduced by the author.

The intricate mathematical expressions presented by Professor Mononobe are perhaps justified in that they provided one means for obtaining theoretical surface curves which coincided almost exactly with the experimental results. However, it seems that the same effect could have been accomplished by much simpler methods. If, in outlining surface curves by computing distances between successive depths, there would be a loss of precision and time, then the author's functions would be of distinct help, however, since such is not the case it seems that nothing in particular is gained for any engineer or student of hydraulics to familiarize himself with the mathematics presented so that he can use the author's methods with confidence.

From the standpoint of presenting the subject of back-water and drop-down curves to those unfamiliar with the hydraulics of varied flow, the author's discussion of the fundamentals could be improved if it were pointed out how the momentum principle applies and how it can be used as a basis for the development of the fundamental equations just as readily as

Bernouilli's theorem. For instance, since the acceleration (or retardation) of the water at any point is equal to $\frac{V\,dV}{dx}$, it can be readily shown that the change in velocity head with respect to distance is equal to the ratio of the water acceleration to the gravity acceleration. From Equation (3) it can be seen that this ratio is equal to the difference between friction slope and slope of the water surface. The acceleration term is very useful in indicating what must be done with eddy and turbulence losses for flow at a decreasing velocity. The writer wonders whether there was any need to take account of such extra losses in the author's experimental data on back-water. No definite mention of it is made in the discussion of the test results.

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DISCUSSIONS

ANALYSIS OF VIERENDEEL TRUSSES

Discussion

BY MESSRS. LOUIS BAES, E. C. INGALLS AND RALPH B. PECK, HAROLD E. WESSMAN, AND A. FLORIS

LOUIS BAES,[39] ESQ. (by letter).[39a]—The author develops the equations of his problem very skilfully, but he states, under the heading "Conclusions", that his method is based upon the principles of virtual work. The writer does not discern anywhere in the paper that these principles are involved. On the other hand, Equations (1), (21), (27), (33), and (38), which are the fundamental formulas, are merely the Bresse formulas, relating to the deflection of the mean fiber of curved members, applied to the case of rigid frames (polygonal members).

Under the heading, "Introduction to Stress Analysis", the author states that his aim is to describe a method of analysis less laborious than others actually in use in the United States. In Belgium, where a great number of Vierendeel trusses have already been built, methods of analysis much simpler than that of the paper are being used. Among others, there is a method developed by Professor Vierendeel himself and another by Professors Keelhoff and Magnel. These methods are exact, in the usual sense of this word, for symmetrical trusses; they include simplifying assumptions for non-symmetrical trusses, which is also true for the author's method.

The simplifying assumptions in the paper involve the reasonable selection of the values of the two coefficients which are, in fact, the ratios between the bending moments taken by the upper chord and by the lower chord. The author rightly shows that the value reasonably assumed for these ratios only slightly effects the results. This is only true within the customary limits, but not in a general way. In fact, the author recalls that Professor Vierendeel uses the assumption that,

$$\alpha = \beta = \frac{I'_c L}{I''_c L_l} \qquad (154)$$

NOTE.—The paper by Dana Young, Assoc. M. Am. Soc. C. E., was published in August, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. L. J. Mensch, A. A. Eremin, Leon Blog, A. W. Fischer, and L. C. Maugh; and January, 1937, by John E. Goldberg, Jun. Am. Soc. C. E.

[39] Prof. of Civ. Eng., Univ. of Brussels, Brussels, Belgium.

[39a] Received by the Secretary October 19, 1936.

and that this assumption greatly simplifies the work. The assumptions generally made in Belgium refer to the location of the point of contraflexure in the vertical members of the truss.

The methods by Professor Vierendeel and Professors Keelhoff and Magnel assume the location of these points of contraflexure to be known and, this location being given, make the truss statically determinate in cutting it into two great combs separated from each other at the points of contraflexure in each vertical. This leads them to one equation which is the equivalent of Equation (53), but instead of the long Equations (66) and (68), they obtain only simple formulas. This results from the fact that the author does not take advantage of the location of contraflexure points, except in the case of symmetrical trusses. Consequently, he finds himself confronted by three systems of equations of the type of Equations (53), (66), and (68)

These three systems are rather long and difficult to solve and require (as has been recognized in Belgium) either very long direct numerical computations, or the use of the method of successive approximations (see Example 4 or Example 5). In fact, the results are derived from the difference between two very great numbers which differ slightly, so that if one does not compute by successive approximations, one might obtain entirely erroneous numerical results. Thus, the method suggested by Professor Young seems to be longer than those developed by Professor Vierendeel or by Professors Keelhoff and Magnel.

The writer has proved[40] that the location of the points of contraflexure in the verticals can be found with great precision by a single formula, which has been checked by more than fifty photo-elastic tests conducted on fifteen models of non-symmetrical trusses. These tests cover a considerable field, extending beyond all the needs of bridge construction, and show the points of contraflexure in the verticals remarkably well. They reveal that these points are actual physical elements on which it is logical to base the method of analysis.

Finally, the writer has shown[40] that Kriso's method may be generalized to all cases. In this method, contrary to what has been done by Professor Vierendeel or by Professor Keelhoff, the system is made statically determinate by cutting a section in one of the chords in each panel. This generalized method leads to one equation for every group of two or three successive panels separated by two successive verticals. The other equations are elementary.

The only long computation is in the solution of the system of these special equations, which are equal to the number of panels. This solution, however, does not involve any difficulty and the equations may well be compared to the three-moment equations of the continuous beams.[41] Each unknown is obtained by an expression in which all the terms have the same sign. This is very important in that it permits computation by slide-rule. On the other

[40] "La Poutre Vierendeel—Généralisation de la methode de calcul par ouverture des mailles par sectionnement d'une des membrures—Controles par la photo-élasticité", par L. Baes. *l'Ossature Metallique* (Brussels, Belgium), No. 10, October, 1936.

[41] "Calcul d'une poutre élastique reposant, sur des appuis inégalement espaces", par M. Clapeyron, *Comptes Rendus* des l'Académié des Sciences, Paris, Tome 45, 1857, pp. 1076 à 1080.

hand, it is impossible to make any error in sign. In fine, it must be noted that by this method the designers very easily, can determine the influence lines of all the variables in the problem. For the ordinary cases of bridges, influence lines show that computations may be limited to the case of one or two conditions of loading.

To the writer, this is the shortest method; it is "exact" for symmetrical trusses, and almost "exact" for the others since from experimental proofs[40] of this essential fact, the location of the points of contraflexure is known quite accurately.

E. C. Ingalls,[42] Esq., and Ralph B. Peck,[42] Jun. Am. Soc. C. E. (by letter).[42a] —It is unusual to find a paper so carefully prepared that the derivations can be followed step by step without need for pencil and paper. The author is to be complimented for the excellent presentation of the development of his working equations.

The writers have used Professor Young's method to analyze the truss shown in Fig. 25. The relative moments of inertia assumed are indicated at

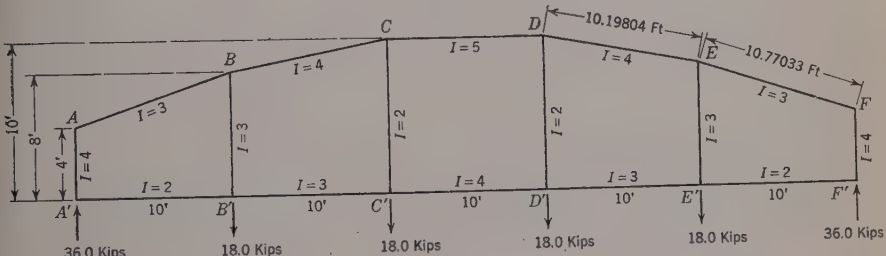

Fig. 25.

the mid-point of each member. The quantities, α and β, were each assumed to unity. The symmetry of both the truss and the loading made the labor involved comparatively slight.

TABLE 2.—Comparison of Results by Slope Deflection and by the Young Method

Quantity	Slope deflection	Young	Quantity	Slope deflection	Young	Quantity	Slope deflection	Young	Quantity	Slope deflection	Young
H_{1w}	33.7	34.2	$M_{B'B}$...	−60.7	−58.4	M_{BC} ..	+16.0	+16.1	$M_{B'C'}$.	+17.9	+18.5
H_{2w}	15.5	15.1	$M_{CC'}$...	−13.7	−13.5	M_{CB} ..	+25.0	+24.7	$M_{C'B'}$.	−22.6	−22.2
H_{3w}	2.7	2.7	$M_{C'C}$ ..	−13.6	−13.4	M_{CD}..	−11.4	−11.1	$M_{C'D'}$.	− 9.0	− 8.7
$M_{AA'}$...	−73.0	−75.4	M_{AB}...	+73.0	+75.4	$M_{A'B'}$.	+61.8	+61.6	V'_1....	24.5	25.8
$M_{A'A}$...	−61.8	−61.6	M_{BA}...	+47.6	+46.1	$M_{B'A'}$.	+42.9	+39.9	V'_2....	13.9	13.9
$M_{BB'}$...	−63.6	−62.2	.....	....	....	......	....	....	....	....	....

To obtain a more correct analysis, involving no assumption of α and β, the writers analyzed the truss using the slope-deflection equations. Although the solution of the simultaneous equations makes the labor great,

[42] Troy, N. Y.

[42a] Received by the Secretary December 2, 1936.

so that the writers do not recommend the slope-deflection method as a practical means of solving this truss, the problem is more general than any which they have seen solved by slope deflection. It includes a modified form of the bent equation and certain relations between the R-values for each panel that would seem to justify its consideration as an alternate method of solution for the Vierendeel truss.

A detailed discussion of alternate methods is beyond the scope of discussion; but a comparison of results obtained by the slope-deflection method for the truss of Fig. 25, with those obtained by Professor Young's method, assuming $\alpha = \beta = 1$, is given in Table 2. The agreement is quite satisfactory.

HAROLD E. WESSMAN,[43] ASSOC. M. AM. SOC. C. E. (by letter).[43a]—There have been considerable additions in recent years to the literature of multistory rigid frames, namely, the steel or reinforced concrete building skeleton. Little has been written in the United States, however, about the multi-panel rigid frame known as the Vierendeel truss. This paper, therefore, is of some value not only because of the contribution made by the author, but also because of the discussion which it has provoked.

As the author states, the paper is restricted to considerations of stress analysis only. Hence, discussion should confine itself primarily to analysis. Nevertheless, it is worth noting that, from the practical viewpoint, the Vierendeel truss is one of those structures in which it is difficult to separate analysis and design. It is simple enough to assume values for the relative stiffness of chords and web members in order to make an analysis of the shears, bending moments, and direct stresses; but when the designer proportions the various members on the basis of this analysis and in accordance with limiting unit stresses established by specification, and then makes a new analysis based on correct values of relative stiffness, he finds that the resulting shears, moments, and direct stresses may be radically different from those obtained in the first analysis. The normal procedure is to revise the design in accordance with the new values, but then, when the process of analysis is again repeated, considerable variations may again be found. In other words, the analysis is so sensitive to changes in sections, that it is almost impossible to obtain the final design in one or two trials, unless it is recognized that excess section must be used for some members.

Here, again, the designer faces the questions: "From where does one start? Can one use the same limiting unit stresses in all members or must one 'waste' material in order to get a design that conforms to the analysis?" This particular problem is another one which emphasizes the need for research in preliminary design methods, research in which analysis and design are more closely correlated.

From the standpoint of elementary mechanics, the author's paper is of value in illustrating the statics of free body diagrams. From the standpoint of advanced structural analysis, the paper is of value in illustrating the application of virtual work equations to satisfy the condition that there

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[43a] Received by the Secretary December 4, 1936.

must be no relative translation, horizontally or vertically, or any relative rotation at a cut section in a continuous structure. The writer feels, however, that the author's presentation could have been improved at certain points, although he realizes the limitations imposed by condensing a paper. For example, Equation (1) would be much clearer if the author had noted that the origin of co-ordinates for the first integral on the right side is at Point B (see Fig. 2(a)) and that the origin of co-ordinates for the second integral is at the upper right-hand corner. It is also worth noting that, although the displacement of Point B is found with respect to Point A, the absolute base of reference is the upper left-hand corner in Fig. 2(a). By fixing this corner and drawing all moment diagrams as if there were two cantilevered members, one extending down to Point A and the other extending to the right and then down to Point B, the basic equation may be easily visualized in terms of area moments.

The same criticism also applies to Professor Young's treatment of the unsymmetrical case. In Equation (21), origins of co-ordinates are not the same for each integral. In connection with this equation, it may be stated that students using area moments blindly are prone to omit the first term, $E\,\theta_c\,(D_b - D_a)$. In solving for the movement of Point D relative to Point C, the rotation of the end, C, when it is not on the same level as Point D must be considered, of course.

In the development of his equations the author emphasizes one method for the analysis of rigid frames. It is a method which is based fundamentally on setting up independent equations for all the unknowns and solving them simultaneously. Presumably, it is an "exact" method; nevertheless, in attacking the unsymmetrical case, certain approximations must be made in order to obtain a solution readily. These approximations, as the author indicates, are not necessary in the symmetrical case. It is evident from the work done by Professor Young and from the equations set up by various other investigators that the computations necessary in order to reach a final design by this method are over-burdening, especially when several analyses are required.

The writer much prefers another method for the solution of the multi-panel rigid frame. It is moment and shear distribution facilitated by superimposing arbitrary joint rotations in order to hasten convergence. There is nothing new[44] about the method, although it does not seem to be widely known, at least in the accurate sense.

The solution of the symmetrical case (see Example 1) by this method is shown in Fig. 26, in which, consistent with the notation of the paper, M_R = the moment from joint rotation; M_V = the moment from shear; M_B = the balancing moment; M_C = the moment carried over; and, ΣM = the summation of moments. Note the rapid convergence of values. For all practical purposes, the procedure could have been terminated after two cycles and the results would be sufficiently accurate. If the arbitrary joint rotations had not been superimposed upon the shear translations, however, the convergence would be much slower.

[44] "Continuous Frames of Reinforced Concrete", by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., pp. 229–233; in particular, footnote at bottom of p. 233.

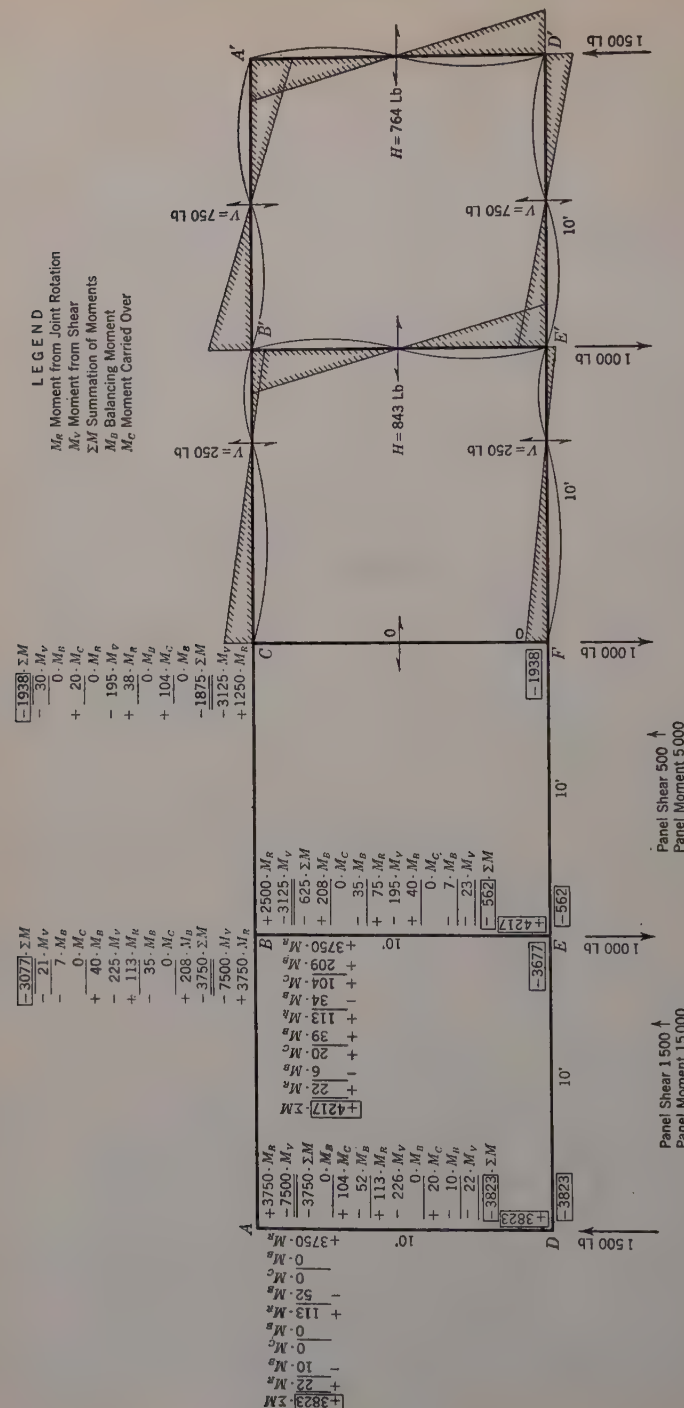
LEGEND
M_R Moment from Joint Rotation
M_V Moment from Shear
ΣM Summation of Moments
M_B Balancing Moment
M_C Moment Carried Over
H = 764 Lb
H = 843 Lb
V = 750 Lb
V = 250 Lb
1 500 Lb
1 000 Lb
Panel Shear 1 500
Panel Moment 15 000
Panel Shear 500
Panel Moment 5 000

Fig. 26.

The process will not be explained in detail in this discussion. It is worth noting, however, that a sketch of panel distortions due to joint translations acts as a guide in indicating the sense of the arbitrary joint rotations. It is not necessary to impose equal rotations at all joints; it is not necessary to make all of them of the same sense; moreover, one does not need to know the actual rotation as long as the arbitrary moments at each end of each member due to rotation bear the proper ratio to one another. If the ratio is correct, geometrical continuity is preserved.

Fig. 27 shows the results from three cycles of computations when this process is applied to the author's unsymmetrical case (see Example 3). All calculations were made on a slide-rule.

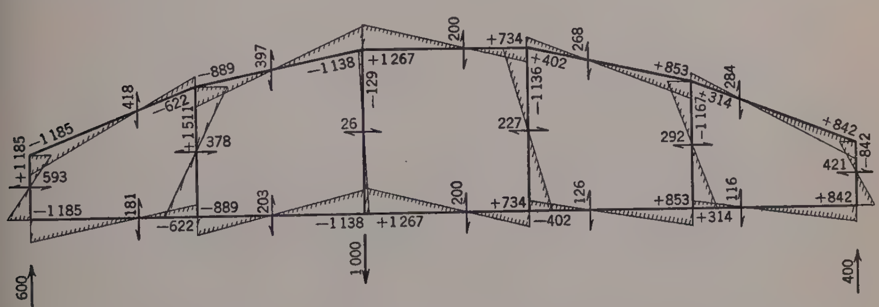

FIG. 27.

This paper demonstrates to the writer that a slide-rule, combined with the proper method of attack, is a rather useful and universal instrument; it is much less expensive than a computing machine and its effect on the eyes not nearly so severe as a nine-place logarithmic table. Even if the author's results are obtained by the so-called "exact" solution of simultaneous equations, it must be kept in mind that his work also involved certain approximations. Moreover, when certain values depend upon small differences between two large values, it is evident that the large values must be determined with extreme precision to a great number of significant places in order to obviate errors in the final result. That is one serious drawback to methods such as that used by Professor Young.

As long as procedure is scientific, however, the most important matter is not "what method was used", but rather, "how shall one interpret the results in the light of the actual design."

The alternate method cited by the writer[38] may also be used in the analysis of wind stresses in multiple-story buildings. It greatly facilitates convergence and obviates the need for extending a mathematical series to obtain the final answer.

A. FLORIS,[45] ESQ. (by letter).[45a]—The classical methods used by the author in the analysis of the Vierendeel truss will undoubtedly divert the attention of engineers, somewhat, from the so-called arithmetical methods which

[45] Dipl.-Ing., Los Angeles, Calif.

[45a] Received by the Secretary January 12, 1937.

have almost dominated the technical literature since the introduction by Hardy Cross, M. Am. Soc. C. E., of the moment distirbution method.

The author sidesteps, wisely, the question of the merits of this type of structure. This question, however, is of great importance in practical problems involving safety and economy. Unfortunately, the Vierendeel truss is fundamentally inferior to trusses without diagonals, for the following obvious reasons. In the ordinary truss the primary direct stresses can be calculated with great accuracy. The analysis of the secondary bending stresses, of course, is less accurate. These stresses, however, can be reduced to a minimum in a properly designed structure. In the Vierendeel truss, on the other hand, the bending stresses become primary stresses. Consequently, the structure will be more expensive and, because of the approximate nature of such an analysis, less safe stresses are allowed unless a larger factor of safety is adopted. In general, it is more expensive to transfer loads to the foundation by means of members subjected mainly to bending than by means of members subjected preponderately to direct stress.

Since the first analysis of the truss under consideration was given by Professor Vierendeel[46], the problem has attracted, widely, the attention of theorists. This is obviously due to the desire to simplify the complicated, although not necessarily difficult, calculations. Theoretically, the analysis of statically indeterminate structures does not present essential difficulties. Any one familiar with such an analysis can write down the necessary equation with ease. However, the time consumed in the derivation of these expressions and their subsequent tedious numerical evaluation are serious obstacles to their use in practice.

Following the trend for greater simplicity and expediency the author gives an approximate (and, for practical purposes, sufficiently accurate) method of analyzing the Vierendeel truss. The writer does not intend to enter into a detailed discussion of this excellent paper. For a proper appreciation of the work done and the efforts made in this direction by the author and others, the available extensive literature on the subject should be studied carefully.[47]

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[46] "Longerons en treillis et longerons à arcades", par Arthur Vierendeel, Paris, 1897.

[47] "On the Theory of Trusses Without Diagonals", by G. P. Peredery, Moscow, 1905 (in Russian); also "Reinforced Concrete Bridges", by G. P. Peredery, St. Petersburg, 1912, p. 166 (in Russian); "Trusses Without Diagonals, Their Analysis and Application to Steel and Reinforced Concrete Structures", by J. Podolsky, Moscow, 1909 (in Russian); also, *Beton-Kalender*, 1929, Pt. I, 290; "Beitrag zur Berechnung von Vierendeelträgern", von A. Ostenfeld, *Beton und Eisen*, 1910, p. 30; also, "Die Deformationsmethode", von A. Ostenfeld, Berlin, 1926, p. 86; "Beitrag zur Theorie der Vierendeelschen Träger", von H. Marcus, *Armierter Beton*, 1910, p. 208; "Einflusslinien für die Berechnung paralleler Vierendeelträger", von W. St. Ritter von Balicki, Berlin, 1910; also, "Vierendeel-Träger mit Parallelen Gurtungen", von E. Reich, Wien, 1911; "Gesetzmässigkeiten in der Statik des Vierendeel-Trägers", von L. Freytag, München, 1911; "Der Pfostenträger mit ungleichen Gurtungen", von L. Mann; in Festschrift für H. Müller-Breslau, Leipzig, 1912; "Die Berechnung der Pfostenträger", von Otto Mohr, *Der Eisenbau*, 1912, p. 85; also, "Abhandlungen aus dem Gebiete der technischen Mechanik", von Otto Mohr, Berlin, 1914, p. 506; "On the Analysis of Trusses Without Diagonals", by J. Podolsky, *Engineer* (Kiev), 1913, No. 7 to 10 (in Russian); "Die Berechnung der Rahmenträger", von F. Engesser, Berlin, 1913; "Die Berechnung statisch unbestimmter Tragwerke nach der Methode des Viermomentensatzes", von Friedrich Bleich, Berlin, 1918, p. 125 "Die Berechnung der Rahmenträger", von Tschalyscheff, *Der Bauingenieur*, 1922, pp. 208 and 244; "Die statisch unbestimmten Systeme des Eisen-und Eisenbetonbaues", von Friedrich Hartmann, Berlin, 1922, p. 169; "Statik der Vierendeelträger", von K. Kriso, Berlin, 1922; "Calcul des Constructions Continues à Eléments Droits", par P. Thomas, Paris, 1925, p. 57; "Berechnung Statisch Unbestimmter Biegefester Stab-und Flächentragwerke", von Peter Pasternak, Zürich, 1927, Pt. I, p. 240. "Analyse statique des poutres Vierendeel", par B. Enyedy, Paris, 1928; "Die Statik des ebenen Tragwerkes", von Martin Grünig, Berlin, 1928.

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DISCUSSIONS

SIMPLIFIED METHOD OF DETERMINING TRUE BEARINGS OF A LINE

Discussion

By Messrs. J. C. Pinney, R. L. Vaughn, and John C. Penn

J. C. Pinney,[20] M. Am. Soc. C. E. (by letter).[20a]—The determination of the true meridian by direct solar observation is a practical necessity in ordinary surveying work, and any method which will reduce the time necessary for computing the field observations without sacrificing accuracy is worthy of consideration. Several formulas have been developed for computing the azimuth, Z, of the sun from the spherical triangle. The four which the writer recalls give the solution in terms of cos Z, vers Z, cot $\frac{Z}{2}$, and sin $\frac{Z}{2}$, and from these terms an equation can be developed for other trigonometric functions. With the ordinary instruments and methods used by the land surveyor, about 1′ in azimuth is the limit in accuracy, and computations carried beyond 0.1′ are futile.

Mr. Inch has presented a method for solving the cosine formula arithmetically in a much shorter time than a regular arithmetic solution would take without the use of a computing machine. This is accomplished by presenting the major computations pre-worked in the form of Table 1. The writer's main criticism of this table is that a straight-line interpolation is used for determining the constants, A and B, between whole degrees. For certain values of the altitude and latitude, this may lead to appreciable errors.

The cosine formula, Equation (1), is usually assumed as being adaptable to natural functions rather than to logarithms. The writer, however, believes that when properly arranged it is very susceptible to logarithms, and he presents herewith the solution of Example 1 by logarithmic computations of both the cosine and versine formulas, comparing the number of operations

Note.—The paper by Philip L. Inch, Assoc. M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. Earl F. Church, Paul E. Wylie, James B. Goodwin, C. H. Swick, Philip Kissam, and George D. Whitmore; December, 1936, by Messrs. O. H. Chilton, Chalmers C. Schrontz, Frank M. Johnson, Walter H. Starkweather, and C. I. Day; and January, 1937, by Messrs. L. McKee, F. J. Duarte, and Leonard C. Jordan.

[20] Asst. Cadastral Engr., U. S. Biological Survey, Washington, D. C.

[20a] Received by the Secretary November 27, 1936.

involved in these formulas and in Mr. Inch's solution. For convenience of reference the versine formula is given:

$$\text{vers } Z = \left\{ \sin \left[90 - (h + \phi) \right] + \sin \delta \right\} \sec h \sec \phi \ldots\ldots(10)$$

Example 6.—Using Equation (1), with north as the zero azimuth, and correcting the observed h for refraction and parallax:

$$\begin{array}{llrl}
\text{colog cos } 0.044128 & \ldots h = & 25^\circ\, 23.'6 & \ldots \text{log tan } 9.676408 \\
\text{colog cos } 0.108851 & \ldots \phi = & 38^\circ\, 53.'7 & \ldots \text{log tan } 9.906741 \\
\text{log sin } 8.258190n & \ldots \delta = & -1^\circ\, 02.'3 & \\
\hline
\text{log } A \quad 8.411169n & A = & -0.02578 & \\
 & B = & 0.38296 & \qquad 9.583149 \\
 & A{-}B = & -0.40874 & \\
 & = & \cos 245^\circ\, 52.'5 & = \text{S } 65^\circ\, 52.'5 \text{ W}
\end{array}$$

Example 7.—Using Equation (10) with south as zero azimuth, and correcting h as in Example 6:[21]

$$\begin{array}{lrrl}
 & h = & 25^\circ\, 23.'6 & \ldots \text{log sec } 0.044128 \\
 & \phi = & 38^\circ\, 53.'7 & \ldots \text{log sec } 0.108851 \\
\hline
 & \text{sum} = & 64^\circ\, 17.'3 & \\
\text{nat sin } \quad 0.43384 \ldots & \text{co-sum} = & 25^\circ\, 42.'7 & \\
\text{nat sin } -0.01812 \ldots & \delta = & -1^\circ\, 02.'3 & \\
\hline
\text{Algebraic sum } \quad 0.41572 \ldots & & & \ldots \text{log } 9.618801 \\
 & & & \text{log vers } 9.771780 \\
 & Z = & 65^\circ\, 52.'5 & = \text{S } 65^\circ\, 52.'5 \text{ W}
\end{array}$$

Although the most rapid method for obtaining the final result depends largely upon the computer and the method of procedure with which he is best acquainted, it may be worth while, nevertheless, to compare these three solutions in respect to the number and kind of operations required in each. These operations may be classified as: Arithmetic multiplication and division, arithmetic addition and subtraction, and references to tables. Of these, multiplication and division are to be particularly avoided as tedious and, therefore, subject to errors.

TABLE 7.—COMPARISON OF WORK INVOLVED IN SOLUTIONS

Method	Example 6	Example 7	Inch method
Addition and subtraction	0	0	5
Multiplication and division	4	5	3
Table reference	7	7	2

The comparison (adding one more addition and one more table reference to the cosine and versine methods than is shown in Examples 6 and 7 because Table 1 includes refraction correction) is shown in Table 7. It is assumed for the cosine formula (Example 6) that the cosine and tangent combined

[21] This solution is taken from "Azimuth" by the late George L. Hosmer, M. Am. Soc. C. E., and the arrangement is taken from Form Bi–1096 of the U. S. Biological Survey.

will constitute only one table reference as the logarithmic sines, cosines, tangents, and cotangents are contained in a single table.

As compared with the method given by Mr. Inch the writer believes either of the other two methods are preferable, because of the five arithmetic multiplication and division operations alone. The cosine method seems to have a very slight advantage over the versine method only if the computer is able to write down the co-log cosine directly from his reading of the log cosine. Even in this case there is a fertile field for error. This same objection is also true for the versine method in case the computer has no table of log secants (= colog cosines). If the computer is equipped with a handbook[21] containing consecutive, five-place tables of log secants, natural sines, logarithms of numbers, and log versines, the versine method is considerably shorter.

The foregoing discussion applies primarily to conditions where the computations are to be made in the field, or under circumstances where a computing machine is not available.

R. L. VAUGHN,[22] M. AM. SOC. C. E. (by letter).[22a]—Of all methods of obtaining a true bearing that by observation of Polaris at elongation is the most simple. It has several disadvantages, among which may be mentioned: (1) In high latitudes it is difficult to observe, and even more difficult to obtain, a desirable degree of accuracy; and (2) in low latitudes, even on clear nights, the star is likely to be obscured by atmospheric haze, the necessary arrangements for illuminating the cross-hairs, the verniers, and the station sighted on entail more or less bother. A fairly accurate determination of the latitude is essential. Lastly, the time of elongation is more than likely to be at a most inconvenient hour. Accurate local time is not indispensable.

The "any hour method" of observation on Polaris is subject to all but one of the foregoing objections, and furthermore requires accurate local time (not standard time). It does possess the advantage that the observation may be made at any time when Polaris is visible.

Methods of obtaining true azimuths or bearings by observation on the sun, of course, are by no means new. To an engineer not particularly versed in field astronomy and who has only occasional need for making an azimuth determination, the method may appear involved and laborious. Such, however, is not the case. Whenever a determination accurate to the nearest minute will suffice, sun observations will be found to be greatly superior to observations on Polaris both as regards speed and as regards convenience.

A number of formulas are available, one of which is given by the author in Equation (1). This formula is well adapted to computations where natural functions and a calculating machine are used. For computations using logarithmic functions a somewhat more convenient formula is:

$$\cot \tfrac{1}{2} A = \sqrt{\sec\ S,\ \sec\ (S - P),\ \sin\ (S - h),\ \sin\ (S - \phi)} \ldots\ldots (11)$$

[22] Cons. Engr., San Francisco, Calif.

[22a] Received by the Secretary January 6, 1937.

TABLE 8.—AZIMUTH COMPUTATION FROM SOLAR
(Latitude, 37° 21′ 00″ N; Longitude, 122° 54′

Description (1)	Set I (Direct) Horizontal angles (2)	Vertical angles (3)
Mean Parallax and refraction	140°–03′	31°–28′–30″ 0°– 1′–20″
True altitude, h		31°–27′–10″
Local (average) time of observation Longitude (hour angle)	9:00 A.M. 8:00 A.M.	
Greenwich (average) time of observation Interval from Greenwich noon	5:00 P.M. 5:00 P.M.	
Corrected declination ±90°	S 11°–52′–24″ ± 90°– 0′– 0″	
North polar distance, P Altitude, h Latitude, ϕ	101°–52′–24″ 31°–27′–10″ 37°–21′– 0″	
2 S	170°–40′–34″	
S North polar distance, P	85°–20′–17″ 101°–52′–24″	
$S - P$	16°–32′–07″	
S Altitude, h	85°–20′–17″ 31°–27′–10″	
$S - h$	85°–53′–07″	
S Latitude, ϕ	85°–20′–17″ 37°–21′– 0″	
$S - \phi$	47°–59′–17″	
log sec, S log sec, $S - P$ log sin, $S - h$ log sin, $S - \phi$	1.09 008 0.01 834 9.90 732 9.87 099	
log cot², (0.5A) log cot, (0.5A)	0.88 673 0.44 336	
0.5A A	19°–48′–45″ 39°–37′–30″	
Azimuth of sun (North = 0°–0′–0″)* Angle from sun to mark†	140°–22′–30″ 140°–03′– 0″	
Azimuth, station to mark†, Set I	0°–19′–30″	
Azimuth, Set II Azimuth, Set III Azimuth, Set IV	0°–19′–00″ 0°–19′–30″ 0°–18′–30″	
Total; divide by four	0°–76′–30″	
Average azimuth, station to mark† Bearing, station to mark†	0°–19′–00″ N 0°–19′–00″ E	

* In the afternoon, the azimuth of the sun is A; in the forenoon, it is 360° A (from south point).
† For example, "East corner of the chimney, gymnasium of the P. Burnett, Jr., High School.

in which A is the angle between the true south and the sun, measured clockwise if the observation was made in the afternoon; counter-clockwise, if the observation was in the morning; P is the north polar distance of the sun (90° plus a south declination, or minus a north declination); h is the altitude,

OBSERVATIONS; SOLUTION OF EQUATION (11)
00″ W; Standard Time Meridian, 120° W)

Set II (Reversed)		Set III (Reversed)		Set IV (Direct)	
Horizontal angles (4)	Vertical angles (5)	Horizontal angles (6)	Vertical angles (7)	Horizontal angles (8)	Vertical angles (9)
320°–38′–30″	31°–46′–30″ 0°– 1′–20″ 31°–45′–10″	321°–16′–30″	32°–06′–0″ 0°– 1′–20″ 32°–04′–40″	142°–13′–30″	32°–33′– 0″ 0°– 1′–20″ 32°–31′–40″
9:00 A.M.		9:00 A.M.		9:00 A.M.	
....		....		....	
S 11°–52′–24″ †90°– 0′– 0″		S 11°–52′–24″ †90°– 0′–00″		S 11°–52′–24″ †90°– 0′– 0″	
101°–52′–24″ 31°–45′–10″ 37°–21′–00″		101°–52′–24″ 32°–04′–40″ 37°–21′–00″		101°–52′–24″ 32°–31′–40″ 37°–21′–00″	
170°–58′–34″		171°–18′–04″		171°–45′–04″	
85°–29′–17″ 101°–52′–24″		85°–39′–02″ 101°–52′–24″		85°–52′–32″ 101°–52–24″	
16°–23′–07″		16°–13′–22″		15°–59′–52″	
85°–29′–17″ 31°–45′–10″		85°–39′–02″ 32°–04′–40″		85°–52′–32″ 32°–31′–40″	
53°–44′–07″		53°–34′–22″		53°–20′–52″	
85°–29′–17″ 37°–21′–00″		85°–39′–02″ 37°–21′–00″		85°–52′–32″ 37°–21′–00″	
48°–08′–17″		48°–18′–02″		48°–31′–32″	
1.10 423 0.01 800 9.90 649 9.87 201		1.12 005 0.01 765 9.90 559 9.87 311		1.14 307 0.01 716 9.90 442 9.87 462	
0.90 073 0.45 036		0.91 630 0.45 815		0.93 927 0.46 964	
19°–31′–15″ 39°–02′–30″		19°–12′–00″ 38°–24′–00″		18°–44′–00″ 37°–28′–00″	
140°–57′–30″ 140°–38′–30″		141°–36′–00″ 141°–16′–30″		142°–32′–00″ 142°–13′–30″	
0°–19′–00″		0°–19′–30″		0°–18′–30″	

of the sun, corrected for parallax and refraction; ϕ is the latitude of the station from which the observation is taken; and,

$$S = \tfrac{1}{2}\,(P + h + \phi)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Whenever S is less than P or h or ϕ, merely subtract the smaller angle from the larger and proceed without regard to the negative algebraic sign.

During the early years of its existence, regulations of the Bureau of Public Lands of the Insular Government of the Philippine Islands required that an astronomical determination of azimuth should be made as a part of each land survey. The solar method was preferred on account of reliability

and convenience. The standard observation consisted of eight pointings on the sun, as shown in Table 8, Sets I and IV being made with the telescope erect and Sets II and III with the telescope reversed, and the *A* vernier being read in all cases. A fairly rapid instrumentman could set up an instrument and take the entire observation in a period of 20 min, or less; a fairly proficient computer could perform all the calculations in an additional 20 min. It cannot be said that the determination of a true bearing was an unduly burdensome task.

The true purpose of the time of observation seems to be misunderstood by some surveyors. Accurate local time is not required for a solar observation. The only purpose for which time is used is to obtain the sun's declination at the time of observation, which requires that the Greenwich time of the observation be known within an accuracy of about 5 min. Within the area of the United States the observer's watch will almost certainly be keeping standard time, and in order to obtain Greenwich mean time it is merely necessary to add 5, 6, 7, or 8 hr to the observed time, depending on which standard time is carried. If the observation is made in some other part of the world, it will be necessary to ascertain in some manner the relation between watch time and Greenwich time, but in no case is the longitude of the place of observation needed. In some cases it will be found that the particular Ephemeris which happens to be available gives the sun's declination for Greenwich mean noon. In this case the calculation of the declination for the time of observation is simple and obvious. It may be found that the declination is given for Greenwich apparent noon, in which case the apparent time of observation may be found by adding or subtracting the equation of time (also given in the Ephemeris) to the mean time.

Solar attachments, and sextant telescopes, contain special arrangements of wires by the use of which the sun may be centered. Ordinary transits are not so equipped and it is necessary to make observations in pairs and use averages, as is indicated in Table 8. This method assumes that the path of the sun is a straight line between the two positions in each set. Such is not the case, but no appreciable error is introduced providing the observations are taken with reasonable speed. However, the telescope should not be reversed between the two pointings which constitute any one "set."

Two methods are available by means of which the necessary readings may be taken. In one method the sun's image and the pattern of the cross-hairs are cast upon a card held about 6 in. from the eye-piece. There will be no difficulty in focusing so as to obtain a sharp image of the sun, but the eye-piece must be especially set to obtain a sharp image of the cross-hairs. With some transits there will be no difficulty about focusing the eye-piece. With other instruments, however, it will be found impossible to obtain a sharp image of the cross-hairs within the limit of motion of the eye-piece. In such instances, the small screw which limits the motion of the eye-piece may be removed, after which the latter may be drawn outward a sufficient distance to give the required sharp image of the wires. Fairly good results may be obtained by this method in the hands of an experienced observer.

However, at the exact instant of tangency of any wire with the edge of the sun, the image of that wire is not visible on the card and in order to get really accurate results the observer must be a good guesser.

A much better method of observation is by the use of a diagonal eye-piece. Most of the better instruments are fitted to receive this simple and inexpensive attachment which consists, essentially, of a mirror and a colored-glass shade. When using the diagonal eye-piece it is not necessary to change the focusing of the wires, and the latter will be visible whenever they are in any position near the edge of the sun. It will be found necessary to remove the attachment each time the instrument is transited.

Observed altitudes of the sun must be corrected for refraction. The correction is always subtractive, and is usually obtained from a table of average refraction values. Theoretically, there should be a correction for parallax because the point of observation is on the surface of the earth, not at its center. Parallax is not of great importance in a solar observation, although it is a serious factor in lunar observations. However, both refraction and parallax are a function of the altitude and may be combined quite simply. Many standard texts give tables of combined refraction and parallax, and the correction should be taken from such a table rather than from one giving refraction only. In this connection it is well to emphasize the fact that the author has incorporated the parallax and refraction correction in his table and when using it the observed altitudes should not be corrected. In fact, attention should be invited to this circumstance in the heading to the tables.

Properly used, the solar method is capable of excellent results. The writer does not approve of reversing the instrument between the two observations constituting a pair, as indicated by the author in Fig. 2; nor should reliance be placed on a single pair of observations. The method used by the Bureau of Lands was found to be quite satisfactory. The results of Sets I and IV, Table 8, should agree within 1 min, as should the results of Sets II and III. The discrepancy between the former and the latter may be considerably wider, reflecting the effect of lack of instrumental adjustment. Taking the average of all four sets will eliminate the effect of such instrumental errors and, of course, will minimize accidental errors of observation.

One instrumental error that cannot be eliminated by the method of reversal, is the effect of error in leveling the horizontal plates. Any error in the determination of the latitude will be reflected to an appreciable extent in the results. However, error from this source may be eliminated by taking one complete series of observations in the morning and another in the afternoon.

The method of dealing with horizontal angles shown by the author in Fig. 1 is correct; but with the sun in a different relative position, it might cause confusion because of the necessity of measuring angles in a counterclockwise direction. Adherence to the following rule will avoid all difficulty: Set the horizontal plates so that they read 0° 00′ when pointing along the line the direction of which is to be determined. Read all horizontal angles to the sun in a clockwise direction, from 0° to 360°. Compute the azimuth of the sun

(not its bearing). Subtract the observed horizontal angle from the computed azimuth of the sun, adding 360° to the latter if necessary. The remainder is the desired azimuth of the line.

Table 1 of the paper certainly represents an impressive mass of labor, and the publication of the complete table would be a service to the profession which should be encouraged. It entails double interpolation: One to reach the values for the actual latitude, and another to reach the actual altitude. Interpolation by second differences must be resorted to in order to achieve an accuracy in the computations commensurate with the precision of the method. Experience has shown that it is almost impossible to publish such a table for the first time without some errors. The possibility of such errors should be guarded against when using the table.

The formulas given in this discussion apply equally well to any other heavenly body. The Nautical Almanac gives the necessary data concerning the planets and a selected list of fixed stars. The only difficulty is to identify the particular body under observation. Star charts are available by means of which the fixed stars may be identified. Planets are most easily identified by computing the approximate elevation and time of transit across the local meridian. In some locations and under some atmospheric conditions, it may be desirable to determine azimuth by observation on some object other than the sun or Polaris In case a planet is observed, which is closer to the earth than the sun, the parallax correction should be given due consideration. Observations on the moon are not likely to give very good results, and this body is best avoided.

JOHN C. PENN,[23] M. AM. SOC. C. E. (by letter).[23a]—The most favorable time for observations on the sun is when it is on or near the prime vertical; that is, when it is east or west, or nearly so. The azimuth of the sun is then ± 90° (0° is assumed to be north). Any formula that solves the azimuth by means of its "cosine" necessitates entering a logarithmic table or tables of natural functions where the interpolation is most difficult and most inaccurate. In the United States, most work will be done in the field when the sun's declination is north (that is, in the time between March 21 and September 21), and with a lower limit for altitude of 25° (because of uncertainty in refraction correction below that figure), azimuth will fall between 80° and 100°, and slide-rules, tables, and any other arrangements for using cos Z will not be accurate.

Any astronomical method for finding azimuth requires the nautical almanac, an ephemeris, or a similar table. The choice of the year 1910 in the example in Mr. Inch's paper is unfortunate. The astronomical day, like the civil day, has begun at midnight ever since 1925. The American Nautical Almanac now (1937) contains tables for refraction, parallax, etc.

The solar ephemerides, as published by instrument companies for use with solar attachments, contain tables for refractions, but care must be taken in

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[23a] Received by the Secretary, January 14, 1937.

not using these refraction corrections for the setting of the attachment. Refraction corrections involve the latitude of the place and cannot be applied as corrections for an observed altitude.

There are several other formulas available, and surveyors are not necessarily limited to observations on the sun for the determination of meridian. Navigators use,

$$\cos\frac{Z}{2} = \sqrt{\cos S \cos (S - P) \sec \phi \sec h} \quad \ldots\ldots\ldots\ldots (13)$$

in which Z = the azimuth of a heavenly body (star, planet or sun); $S = \frac{1}{2}(h + \phi + P)$ in which h = corrected altitude; ϕ = latitude; and P = polar distance ($90° \pm$ declination). For those not familiar with "secants," this formula may be written:

$$\cos\frac{Z}{2} = \sqrt{\frac{\cos S \cos (S - P)}{\cos \phi \cos h}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Equation (14) solves for azimuth in terms of $\frac{Z}{2}$ and involves logarithmic tables in the vicinity of 45° where the values for cosine mean something.

It is a pity that the present American Nautical Almanac gives the azimuth of Polaris for various hour angles of Polaris only to the nearest tenth of a degree. Older almanacs and the American Ephemeris give azimuths of Polaris to 0.1′. If the American Nautical Almanac gave the same consideration to surveyors as it now does to aviators, the necessity for tables and slide-rules would be entirely obviated and, incidentally, there would be no need for an accurate map for the determination of latitude, as both azimuth and latitude can be determined by an observation on Polaris with very little arithmetic.

If the table suggested by Mr. Inch is to be of any value to a surveyor it must be well understood that the surveyor goes into the field with the following equipment:

(*A*) The table itself, extended to allow for better interpolation;

(*B*) Instructions for use of this table to include: (1) A bold statement that altitudes need not be corrected for parallax and refraction; (2) a similarly strong statement that if the declination of the sun is north, $A \sin \alpha$ and B are subtracted (the smaller from the larger); (3) if $A \sin \alpha$ is the larger, the bearing of the sun will be from the north, and if B is the larger, the bearing is from the south; and (4) if the declination is negative, the bearing is from the south;

(*C*) A table of natural sines;

(*D*) An almanac for the year;

(*E*) A watch correction within a few minutes; and,

(*F*) A quadrangle sheet of the U. S. Geological Survey, or a similar map for the determination of latitude. (Perhaps this map could be left in the office.)

Some of these items are facetious, of course. A little knowledge of trigonometry will eliminate Items B (2), B (3), and B (4), but to one who is not "on his toes" in the mathematics of surveying and the bearing should prove to be 90° ± 20′, these very rules will save many an anxious moment.

The writer would rather carry an almanac, a log table, with the value of cos 0.5 Z neatly printed inside the cover, and a small pad of paper, for meridian determinations during the day. The solution of Example 1 of the paper, by Equation (11) is as follows: Refraction plus parallax correction is 0° 1′ 54″; corrected altitude equals 25° 23′ 36″.

$$
\begin{array}{rll}
P = & 91^\circ\ 02'\ 16'' & \\
h = & 25^\circ\ 23'\ 36''\ \log\sec & 0.04413 \\
\phi = & 38^\circ\ 53'\ 40''\ \log\sec & 0.10885 \\
2\,| & \underline{155^\circ\ 19'\ 32''} & \\
S = & 77^\circ\ 39'\ 46''\ \log\cos & 9.32974 \\
S - P = & 13^\circ\ 22'\ 30''\ \log\cos & \underline{9.98835}\ \text{(always positive)} \\
 & & 2\,|\underline{19.47107}
\end{array}
$$

log cos 0.5 Z = 9.73554 = 57° 3′ 4″; and Z = 114° 6′ 8″.

Bearing of line = 180° − (114° 6′ 8″ = S 65° 53′ 52″ W

Given = S 64° 52′ 30″ W

1° 1′ 22″

By table = 1° 0′ 48″

Difference = 0° 0′ 34″

In the determination of azimuth by a time-altitude of the sun, regardless of the method that may be used, some knowledge of astronomy is absolutely necessary. Without a calculating machine, and in the field, logarithms must take preference over natural functions. All methods so far proposed require either one or the other. A table so involved and so limited as all tables must be, can never take the place of the foregoing simple computation.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE MODERN EXPRESS HIGHWAY

Discussion

By Messrs. A. C. Dennis, and J. C. Carpenter

A. C. Dennis,[36] M. Am. Soc. C. E. (by letter).[36a]—Studied in conjunction with a paper by Fred Lavis, M. Am. Soc. C. E., published in 1930[37], the paper by Mr. Noble constitutes an important advance toward the establishment of correct standards for trunk highways. The technical questions of design standards and economic location can safely be left to the decision of engineers, and the broader question of organization and financing may also be properly the concern of engineers even if the ultimate decision is the responsibility of others. If there is the possibility that certain trunk highways may be constructed, with no addition of taxes to the general public, to provide the users of these highways a safer and faster route free of interruptions at no additional cost to the user, then an engineer's self-interest as well as his duty as a good citizen should lead him to study and discuss this possibility and, if found to be sound, to work for its fulfillment.

The existing highways are, in effect, State toll roads, the toll being paid at the gas station as a State sales tax. These highways have generally been built with local use and convenience as the main objective. Practically all of them are of inadequate capacity for trunk-line traffic and are subject to various traffic interruptions. The result is a condition similar to that of a railway system consisting of many branch lines, but no main line.

Trunk highways are interstate structures beyond the province of the several States, but within the powers of the Federal Government to finance, construct, own, and operate. It appears that a major section of Federal trunk highway of normal cost, carrying about 7 500 vehicles daily on eight traffic lanes, between three or more emergency or parking lanes, may be self-liquidating in that a Federal gas tax collected along the highway at a rate not exceeding that

Note.—The paper by Charles M. Noble, Assoc. M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. Fred Lavis, Joseph Barnett, G. E. Hawthorn, John F. Fairchild, Leslie R. Schureman, and C. H. Purcell; December, 1936, by Messrs. Elmer R. Haile, Jr., H. W. Giffin, and T. T. Wiley; and January, 1937, by Messrs. F. L. McRee, Theron M. Ripley, W. W. Crosby, Richard S. Kirby, Harold M. Lewis, George Conrad Diehl, and William E. Rudolph.

[36] Riverside, Calif.

[36a] Received by the Secretary December 24, 1936.

[37] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1020.

of the local tax, together with revenue from concessions for furnishing service to vehicles and persons, will provide cost of administration, policing, maintenance, and other operating items, as well as interest on the investment with perhaps something toward retirement of investment.

The organization suggested previous to operation, is that a Chief Engineer shall report, possibly, to the Secretary of The Interior, with authority (as far as is practicable for a public servant) similar to that of a Chief Engineer of a railway during location and construction. He should select his staff and be allowed to pay salaries sufficient to secure proper assistants. The Highway Engineer of each State traversed should be a member of his consulting staff.

The first duty of the Chief Engineer should be to select the general route for the highway, establish standards, and make a preliminary estimate of cost and an estimate of immediate and future revenue to determine whether the proposed section is self-liquidating, or whether it will become so in the future. When a proposed section appears to be self-liquidating and when authority has been given to proceed, the Chief Engineer should make location surveys, design structures, acquire rights of way and other necessary lands, prepare specifications, let unit price contracts in sections of several hundred miles each, and supervise the construction.

When a major section of the highway is ready for operation, it should be administered by a Manager reporting, perhaps, to the Secretary of The Interior. He should be responsible for the maintenance of the highway, the leasing and regulation of conclusions, and the enforcement of operating regulations for vehicles and drivers, with the support of Federal Police and Magistrates.

The near future will probably be an unusually expensive period for highway construction, but some dense traffic sections may be economically sound even at excessive cost, whereas the plans for others, not at present self-liquidating, may contain useful knowledge to have available when the next depression again reduces construction costs.

J. C. Carpenter,[38] M. Am. Soc. C. E. (by letter).[38a]—This timely paper covers a subject that deserves most serious consideration by the Engineering Profession. Perhaps the title, "The Modern Express Highway", was selected as a lure to induce some engineers to give thought to the subject of highway safety, who might not be interested in a paper on "The Necessity for Consideration of Safety in Highway Design."

The immediate answer to the problem of highway safety is not the "express highway." Engineers must consider safety in all their designs and cannot wait until funds are available and traffic warrants the construction of divided-lane highways on the large mileage that is now (1937) and will soon be under construction. Mr. Noble's excellent paper covers many features of design that are used in the development of all highway plans and his suggestions are generally applicable to single-lane, as well as to dual-lane, highways. The dual-

[38] Senior Highway Engr., U. S. Bureau of Public Roads, Fort Worth, Tex.

[38a] Received by the Secretary January 2, 1937.

lane highway is undoubtedly superior to the two-way single-lane, and should be used in every instance where the traffic volume and funds available will allow its use. Unless it is plainly obvious that this design is demanded, it will take more than vision or courage to persuade those in authority that the required extraordinary expenditure is justified. Perhaps this paper will provide the data needed by some engineers to convince the authorities that more dual-lane highways are necessary.

As soon as the planning movement, now rapidly gathering headway, has advanced to a point where the results are available, the designing engineers will have reliable and convincing information that will allow them to select, and promote with confidence, the designs that are properly applicable to the conditions dictated by traffic, economic development, and a program based on a correct knowledge of future available funds. The planning program is now in its preliminary stages and the most difficult phases are just ahead. The collection of data is a comparatively simple and easy task, and presents no difficulties not previously surmounted by the engineer, but the co-ordination of these data, analysis, and utilization, to prepare a satisfactory plan that will be so perfect that its general features will be permanent, is an unexplored field and will demand the services of men of broad experience, who have the vision and the courage to look ahead and support their ideas with clearly established reasons. Highway engineers have "followed the old road" and built "within the money" for so long that it will be difficult to break new trails, but those who are privileged to work on this phase of the planning procedure have a large responsibility; and the skill, courage, and good judgment they show, will determine the value of the results to the public and all future generations. One important point will undoubtedly be developed, namely, the funds now collected from highway users are not sufficient to carry on the necessary construction and maintenance operations, including building and maintaining safety in the highways, and funds collected for highway uses should not be diverted for any other purpose.

The safety problem involves many features outside of engineering design and practice. The National Safety Council's three "E"s necessary for safety, are "enlightened engineering", "sagacious education", and "judicious law enforcement." The writer's estimate of the relative endeavor necessary on these three general phases of the safety problem is, engineering, 5%; education, 10%; and enforcement, 85 per cent. Reasons for this estimated rating will be given in the following discussion. This low rating for engineering does not mean to imply that engineers can sit complacently and let the public do the educating and enforcement, but rather that the engineer, with his specialized knowledge and training, while concentrating on the improvement in the engineering phases of the problem, should assume leadership in developing ways and means of providing effective programs for education and enforcement. Education can best be developed through the regular school curricula and through courses in safety required as a penalty for law violation. A regularly scheduled and complete course in all phases of safety, with emphasis on highway safety and practical demonstrations and teaching, should be included as an eighth-grade subject in all the schools of the country.

Drivers' licenses should not be issued to children until they have completed this course and passed a State examination and the enforcement should fit in with this policy. Children of eighth-grade age are just beginning to drive, and development of this policy and legal recognition through the school system and enforcement authorities, will do more to bring about a citizenship thoroughly "safety minded" than any other means of education. Of the adults, the law breakers are the offenders who need to be educated and the majority of them are of the type who will not listen to good advice given as newspaper publicity or through other commonly accepted channels. If they were required to take a carefully planned course in safety before being allowed to drive and to pass a strict examination before having their license returned to them there is no doubt that a large number of violators would be converted into careful drivers.

Enforcement, of course, is a police function, but engineers in private practice and public position can, at every opportunity, stimulate popular support of intelligent enforcement of existing laws and encourage the enactment of necessary additional legislation conforming to the principles outlined in the Uniform Acts of the National Conference on Street and Highway Safety, and can insist on the appropriation of sufficient funds to insure thorough and complete enforcement of all traffic laws.

Highway safety is as much an engineering problem as industrial safety, and engineers can devise methods of control that will be as effective as those so successfully used by the railways and other industries. Careful study of information provided by such organizations as the National Safety Council, the American Automobile Association, and other similar agencies will equip the engineer to lead in community and State safety movements.

The statistics quoted in Mr. Noble's paper emphasize the loss of life and injuries due to traffic accidents. Although it may be that more accidents should be charged to highway design, it is believed that the percentage of the total will not be changed very much. When the figures given are analyzed, it is possible to conclude that the percentage chargeable to design is not as large as might be thought. National Safety Council's "Accident Facts" shows that of the total of 36 100 deaths in 1935, 11 800 were in cities of more than 10 000 population, and 24 300 were on rural highways, which are those under consideration. Of these totals, 35% of the casualties were pedestrians. Although improvement in the design may, accidentally, reduce the pedestrian deaths, it is logical to assume that the majority of them were caused by lack of care on the part of the pedestrian and excessive speed on the part of the driver. A closed right of way on the express highway recommended by Mr. Noble, might keep some of the pedestrians where they belong, but there are other complications, legal and otherwise, which would prevent the strict exclusion of the free and independent public from the public highways of the United States. After deducting 35% for pedestrians, there remains a total of 15 795 deaths which may be more or less remotely charged to defects in design.

The data in Table 6 have been obtained on three highways adjacent to Fort Worth, Tex., covering accidents during the fiscal year ending August 31,

1936. U. S. Highway No. 80 across Palo Pinto County was built in 1924, on what were considered modern standards at that time, but it has many sharp curves, steep grades, and a rough surface 16 to 18 ft wide, with more crown than is necessary. U. S. Highway No. 80, across Parker County, was completed in 1922 as a surface-treated gravel road about 16 ft wide, and approximately one-half of it was reconstructed to a higher standard and wider surface in 1927. State Highway No. 34, from Azle to Jacksboro, Tex., was built in 1934 on excellent alignment, the location having been selected from an aerial survey map, with low degree curves and much better sight distances than on the other two roads; it is surfaced with a bituminous top, 22 ft wide, built to a good section, and generally is in excellent condition. The three sections are about the same length, traverse the same type of terrain, and, except for their age, the surfaces are similar in character. Traffic is heavier in volume on the Parker County Section than on the other two roads, but the difference is not enough to affect the results materially.

TABLE 6.—ANALYSIS OF ACCIDENTS ON EXPRESS HIGHWAYS IN TEXAS

(*a*) GENERAL

Description	U. S. Highway No. 80: Palo Pinto County	U. S. Highway No. 80: Parker County	State Highway No. 34: Azle to Jacksboro, Tex.
When constructed	1924	1922 to 1927	1934
Total accidents	15	45	64
Total killed	2	10	10
Total injured	26	48	49

(*b*) CAUSES OF ACCIDENTS

Description	U. S. Highway No. 80: Palo Pinto County	U. S. Highway No. 80: Parker County	State Highway No. 34: Azle to Jacksboro, Tex.
Drunken drivers	4	...	2
Driver asleep	...	6	7
Blinding lights	...	...	4
Speed	10	15	16
Car defects	...	3	12
Miscellaneous	1	21	23

(*c*) TYPES OF ACCIDENTS

Description	U. S. Highway No. 80: Palo Pinto County	U. S. Highway No. 80: Parker County	State Highway No. 34: Azle to Jacksboro, Tex.
Head-on collision	4	4	8
Overtaking car	3	15	15
Collision with:			
Standing car	6	6	2
Fixed object	6	6	7
Miscellaneous	2	20	34

Table 6 indicates that there are more accidents on the roads of more modern design than on the older types. It is not logical to conclude that one should return to the older design, but one should analyze more carefully all accidents and determine exactly what is the cause; and if design is responsible the engineer must determine what changes are necessary to correct it. It would also seem that the modern design is likely to induce sleep and that about 25% of the accidents are caused by overtaking other automobiles. A change to the dual type will not prevent overtaking accidents, nor will it stop drivers from going to sleep. On all the roads, speed is responsible for a large percentage of the accidents; it is responsible for two-thirds of the total on the oldest road.

It is obvious that a greater volume of accident reports is needed and more complete detailed information on the reports that are made. It is difficult to get the entire "picture", for it is impossible to follow every car and to be on hand to obtain accurate and complete details of each accident. Any information furnished by a driver involved in an accident is usually prejudiced.

The fact remains, however, that engineers need considerably more data than are now available, if they are to design intelligently. Congress, in 1936, appropriated $75 000 for use in investigating conditions and formulating legislation on highway accidents. The Bureau of Public Roads, in co-operation with the Highway Research Council, has appointed an advisory committee to study the causes of highway accidents and conditions contributing to them. These studies will be made in the States that have kept records of accidents and will consist of an analysis of the accident records of a large number of drivers. Publication of the results of this study and wide distribution among engineers will undoubtedly stimulate the more universal collection of accident data; bring about uniformity in the procedure; make available much valuable information for use in design; and result in the enactment of corrective legislation.

This, and other recent papers on this subject[39] set a speed of 100 to 120 miles per hr for the design of highways. Such a high rate is out of all proportion to the capabilities of the present-day driver. Perhaps if there were a uniform drivers' License Law, administered in such a way as to classify all drivers in accord with their skill and dependability, one might allow a select few to pilot automobiles at this rate of speed. Highways designed for this rate of speed will be perfectly suited for the lower speeds that are required, and will provide a factor of safety against obsolescence; but encouraging high speeds will increase accidents, as is emphasized by National Safety Council's "Accident Facts", which shows that at speeds of 20 miles per hr., one accident in 61 is fatal, whereas if the speed is 50 miles per hr., or more, there is 1 death to each 11 accidents. It is believed that practically all the travel over the rural highways of the country can be executed without exceeding 50 miles per hr. and without serious loss of time and money due to this conservative rate. The deaths due to accidents, property damage, and other supplementary losses can undoubtedly be reduced materially if this maximum speed is adopted and the achievement of this control is as much an engineering function as the reduction in degree of curvature, or the determination of the correct rate for superelevation. This same source indicates that in the fatal accidents nine out of ten cars were going straight ahead when the accident happened, thus relieving curvature of the responsibility for a large proportion of the fatal accidents, and indicating that speed was probably at fault.

The dual highway is undoubtedly the correct design where traffic and planning data indicate the location to be of such a permanent nature that the extraordinary expenditure is justified. In a large percentage of the area to be served by an extension of the present highway service the construction of additional routes will better serve the territory than the development of one route to carry a large volume. To illustrate this point, the Fort Worth-Dallas area covers about ten miles in a north and south direction, the centers of the two cities being about thirty-two miles apart. The construction of four 2-lane highways will relieve traffic congestion on the one main-line route now heavily crowded, and these four routes will not cost much more than the dual

[39] See, for example, "Highway Design Applied to the State System", by R. H. Baldock, *Civil Engineering*, October, 1936, p. 643.

type on one route. One, or two, of these routes can be planned for future development as a dual highway, in the manner suggested by Mr. Noble.

The formula for superelevation of surfaces on curves should include a factor for side friction. A superelevation of $1\frac{1}{2}$ in. is about the maximum that can be used. All curves should be superelevated and it would seem to be a simple process to compute the rate between zero for a tangent, and $1\frac{1}{4}$ in. for a 6° curve, using the formulas, including the friction coefficient. There should be very little justification for curves sharper than 6° on any highway that will develop later as a location for a dual highway. If sharper curvature is demanded by the topography it would seem that another route should be investigated. In all highway location the smallest reasonable degree of curvature should be used. It is quite common practice to use a higher degree than necessary, even in new country, due to the fact that the use of a low degree of curvature has not been considered necessary. Many 4° to 6° curves have been used where a 1° curve would fit better, and some 1° curves have been placed where a 10′ or a 30′ curve is easily possible.

Spirals should be used for all changes of direction. The spiral is the natural curve for fast traffic and will tend to keep the cars in the proper lane without cutting corners and, hence, will be a safety measure.

There should be no objection to a reasonable degree of undulation in grade line, provided the sight distance is not reduced. A break of this kind will hide the headlights of an approaching car and will be a welcome relief in some cases. An undulating grade line is almost always less expensive in construction cost, will allow shorter culverts, lower fills which may be sloped so as to avoid the construction of guard rails, and thus be safer and less expensive to maintain. Grades should be fully compensated for curvature. Where dual highways are justified it would seem that a maximum of 5% grade should be demanded, but on the major part of the important highways of the country where the 2-way lanes are to be used, economy would appear to dictate a 7% maximum, unless ice is a factor to be met. In a similar line of reasoning, it seems that a sight distance of 700 ft is a reasonable minimum for the large percentage of the highways but, of course, in all cases, the longest reasonable obtainable distance should be used.

Except in very rough topography, the roadway section should provide a gradually changing slope away from the traveled way, eliminating the defined shoulder point, and providing the maximum of travelable surface on the right of way. All unnecessary obstructions should be removed, but signs are not as much of an obstruction as Mr. Noble indicates. More signs are hit by gunners than by automobiles, and when judiciously used signs are a necessary safety provision. It is impracticable to suspend signs in a windy country. The recently developed center-line reflector button has much merit as a safety device on 2-way traffic lanes.

Guard-rail should be used only where absolutely necessary. For fills less than 10 ft in depth, it is generally more economical to flatten slopes than to use guard-rail, and for deeper fills the additional safety will often justify extra expenditure to eliminate the use of the rail.

There is no immediate possibility of lighting any considerable mileage of rural highways, but with the development of rural electrification the prospect is encouraging. Where conditions demand a dual-lane highway it is logical to assume that lighting will be included.

Mr. Noble suggests the possibility of increasing the thickness of the pavement to provide smoothness. It is the writer's opinion that more detailed study of the supporting sub-grade and provision for adequate control of the moisture content (and, consequently, more uniform supporting power) are the most important phases of design for present study.

This excellent paper will provoke much discussion of value to the engineers of the United States, and the points brought out will enrich the literature on highway engineering. Mr. Noble has performed a valuable service to the profession by raising the question at this time.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

SELECTION OF MATERIALS FOR ROLLED-FILL EARTH DAMS

Discussion

BY MESSRS. JOEL B. COX, STANLEY M. DORE, JOHN E. FIELD, WILLIAM P. CREAGER, AND JOSEPH JACOBS

JOEL B. COX,[24] ASSOC. M. AM. SOC. C. E. (by letter).[24a]—This paper is a greatly needed contribution to the literature on earth dams. Especially to be commended is the clear statement of the requirements of earth suitable for rolled-fill earth dams. The discussion of these requirements is, of necessity, unequal in its satisfaction. The study of grading is an important contribution to current knowledge of the subject, but it must be emphasized that the close relationship between porosity and grading shown by Fig. 1 is obtained from a single series of soils from a comparatively uniform source. The importance of this will be recognized on comparing such an example of Hawaiian laterite as Sample No. 233-*B* in Table 9 which gives a porosity, with the most careful packing, of 64.8 per cent. Its grading curve plotted on Fig. 1 certainly does not indicate the grading as a cause of the high porosity. This porosity is due to the peculiar expanded character of the fine colloidal material in all such soils.

Mechanical analysis cannot be expected to shed much light on this and similar conditions, largely because, by present methods, it is restricted to particle sizes of 0.005 mm and larger.

The excellent treatment of compaction would be improved by a critical discussion of the permissible air content at maximum compaction, which, according to the writer's observation, is a most important quality of suitable soils for rolled-fill construction.

The relationship between permeability and permanence and satisfaction of an earth-filled dam is discussed in this paper insufficiently. The effect of permeability is twofold: (1) The economic loss due to leakage from the

NOTE.—The paper by Charles H. Lee, M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1936, by Messrs. T. T. Knappen, and Paul Baumann; and January, 1937, by Messrs. William C. Hill, A. Floris, and Fred D. Pyle.

[24] Civ. Engr., McBryde Sugar Co., Ltd., Eleele, Kauai, Hawaii.

[24a] Received by the Secretary November 23, 1936.

reservoir; and (2) the effect on the dam itself. The latter is dependent largely on the relationship between the finest fraction of the soil in the dam and the velocity of percolating water. A far higher permeability is safe with a soil composed of sand and gravel than one containing silt and clay.

The author's main proposal that mechanical analysis be used as a preliminary basis for the selection of material, must be carefully covered by reservations, as Mr. Lee does. There is an intimate relationship, of course, between porosity and permeability and the mechanical composition of the material. In any soil containing an appreciable proportion of material finer than 0.005 mm, however, the qualities of the soil are so governed by the finer, colloidal fraction in its composition that mechanical analysis is of little value in interpreting or estimating the observed data. Since permeability and porosity are measured with an ease entirely comparable to that of making accurate mechanical analyses, it is not clear why there is an advantage in using the indirect and remote measurement rather than the direct one as a basis for selection. Table 9 and Fig. 14 give mechanical composition data,

TABLE 9.—GRADING AND PERMEABILITY OF HAWAIIAN SOILS

Sample No.	Sizes, in Millimeters			Porosity (percentage of voids)	Permeability, in gallons per square foot per 24 hours	Sample No.	Sizes, in Millimeters			Porosity (percentage of voids)	Permeability, in gallons per square foot per 24 hours
	10% passing	30% passing	60% passing				10% passing	30% passing	60% passing		
239	(?)	0.003	0.052	64.5	0.00131	248–*A*	......	0.0008	0.013	62.6	0.0102
129	0.0013	0.0046	0.0135	65.6	0.00258	296	0.008	0.0390	0 670	66.0	0.0607
258	0.008	0.0132	0.026	66.5	0.00364	233–*B*	0.022	0.136	0.900	64.8	0.240
333	(?)	0.0143	0.800	60.4	C.00403	.....	......	......	......	....	....

porosity, and permeability for a selected group of seven typical samples of Hawaiian soils, which illustrate the difficulties of using mechanical analysis in comparing materials of unlike origin. The low degree of compaction possible with such soils, the high air content, and the extraordinary elasticity under rolling which makes compaction in an earth dam exceedingly difficult, the impermeability which prevents proper drainage of the down-stream face, and the ease with which the finer fraction is removed by any percolating water, all render these soils extremely hazardous in rolled-fill dam construction, even if some of them fall within the mechanical analysis limits proposed as satisfactory. The lack of relationship between mechanical analysis and permeability is shown

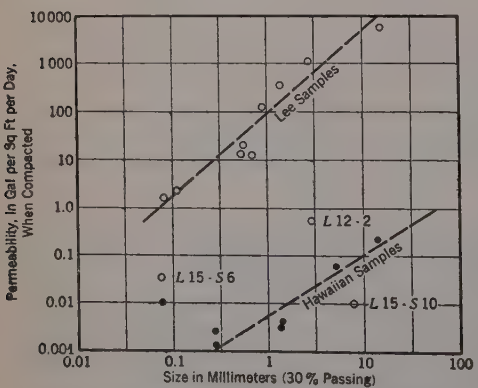

FIG. 14.—RELATIONS BETWEEN PERMEABILITY AND PARTICLE SIZE

not only by these samples, but by such a sample as *L*15-*S*10 of Table 5, the permeability of which is not more than 0.0001th as much as would be estimated from its grading curve in comparison with the others shown.

Fig. 14 shows graphically the enormous difference between soils of entirely different character but with somewhat similar mechanical analysis curves. The fact that the two average lines representing the Lee samples and the Hawaiian samples differ by the ratio of more than 10 000 to 1 in permeability clearly indicates the dangers of too great dependence on such data. The writer believes that although a mechanical analysis may be developed into a useful tool, furnishing a criterion for estimating relative differences between soils of a homogeneous series, it will always be necessary to fall back on direct measurements of permeability, porosity, and compaction in comparing widely different soils.

Mr. Lee's experience with a siltstone soil and his recognition of the undesirability of adobe and lateritic soils for rolled-filled dams is important to the engineer engaged in such work. It is to be hoped that the discussion of this paper will bring forth a practicable proposal for a measurement of the workability of various soils.

STANLEY M. DORE,[25] M. AM. SOC. C. E. (by letter).[25a]—The use of soil laboratory results as aids in determining the suitability of materials for the construction of earth dams is well descrbed in this paper. The stability of an earth embankment depends primarily upon the shearing strength of the materials of which that embankment is built and, as the author points out, high shearing strength in soils exists when there is a proper combination of cohesion and internal friction. Whether or not the cohesion is due to the strength of the minute films of water (as the author contends) many tests in the laboratory and elsewhere bear out the fact that the larger the proportion of a soil that is of the finer grained, or of the clay, size, the greater are its cohesive qualities. Especially in materials of a glacial origin, cohesion and the presence of the finer particles go "hand in hand", and the value that such cohesion reaches depends directly upon the sizes and quantity of these "fines." Thus, without a sample at hand, the cohesion of a material is definitely indicated by a study of the mechanical analysis curve, the number and sizes of the finer particles being shown on the curve.

Separated from the cohesion, the internal friction of a material depends upon the grading or "uniformity coefficient" and upon the shapes of the individual particles, the large number of sizes furnishing more resistance to movement because the smaller particles fill the voids between the larger ones and prevent them from tilting or rocking, and the sharp, angular, cut-clear particles offering much more resistance to movement than rounded, worn, smooth particles. An examination of the mechanical analysis curve would indicate the extent of the grading, and microscopic examinations would reveal the predominating shapes; and from these examinations the internal friction

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[25a] Received by the Secretary December 3, 1936.

of a material is indicated Mechanical analysis curves that are concaved upward show graded material of relatively higher shearing strengths, and curves that are convex upward show ungraded materials of lower shearing strengths. Thus, from the mechanical analysis curves and the microscopic examinations, both factors of the shearing strength can be estimated—namely, the cohesion and the internal friction—and the judgment of the degree of stability of a given material based upon visual examination and "feel" to the hand can be augmented considerably by these estimates.

The shearing strength of a material is increased by compaction. Vibration is the most important factor in securing satisfactory compaction and density, and the author has not directly mentioned vibration as a means for securing compaction. The success of tamping, rolling, or compacting by other mechanical means usually depends to some extent upon the vibration attending these operations, and because of its gerat value in obtaining high degrees of density and compaction, various types of mechanical vibration equipment are supplanting, and will supplant, many of the "pressing or loading" types of compacting machinery now (1936) in use.

Without doubt, the best materials for use in rolled impervious embankments are those which are well graded and which contain fractions of all sizes from pebbles to clay; but the engineer must not lose sight of the fact that rolled fills, satisfactory for the purposes intended, can be built of certain uniform or ungraded materials and that it is often a question of economics whether the more, or the less, preferred materials are used.

In Fig. 4 the compaction curves for loam seem to give the highest densities. Such loams must be well graded, must contain appreciable quantities of pebbly or gravelly materials, and must be consolidated by heavy equipment in order to obtain such high densities. If these loams had contained large quantities of ungraded silts and sand in addition to a small percentage (say, 5%), of organic materials, such high densities would not have been obtained. When compacted in 6-in. layers, with a 5-ton, 40-in., sheepsfoot roller, top soils have dry weights generally within the limits defined by the stipled area in Fig. 15. These values cannot be plotted as a single curve, due to the effect of the variable organic content of the different top soils used on the critical moisture content needed for maximum limits. The limiting curves in Fig. 15 show the general average conditions that yield best results. For instance, in Fig. 15 is indicated the moisture-content, density-compaction curves of loams consisting of a mixture of top soils and subsoils. The organic content in these materials averages 2 to 5% and the diagram shows that with very satisfactory compaction the densities were not greater than about 110 lb per cu ft.

The recommendation of the author for the upper limit of the clay fraction in materials for use in compacted fills is well chosen. Materials that have a clay fraction of about 30 to 35% can be compacted only with considerable difficulty because they cannot be worked satisfactorily due to the fact that in material of this nature enough of the "superfine" and colloidal particles are present to make the mass "sticky", "jelly-like", and resilient. However, the

lower limit is not quite so definite. Materials containing little or no clay can be compacted satisfactorily, provided they are otherwise suitable and that the quantity of the clay fraction necessary is really dependent upon the imper-

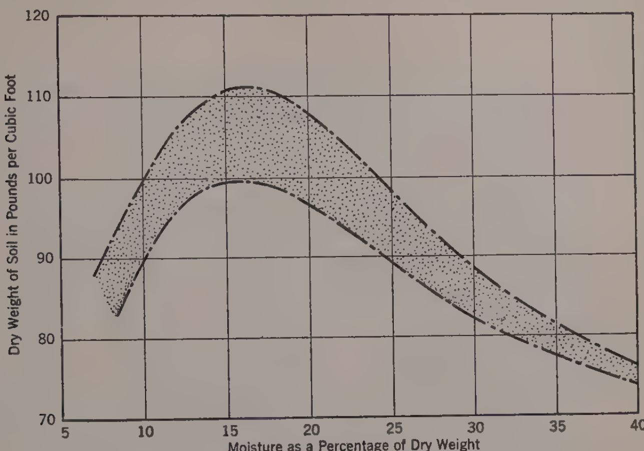

FIG. 15.—FOUNDATIONS TOP-SOILS FOR IMPERVIOUS REFILLS, QUABBIN DAMS

meability requirements and not upon the ease or difficulty with which such material compacts. Of course, the compaction is somewhat easier when a small percentage of clay is present because, when moist, this clay lubricates the mass somewhat; but with sufficiently heavy equipment and the proper moisture contents, materials with little or no clay can be compacted satisfactorily and without undue difficulty or expense. For uniform or ungraded materials Mr. Lee indicates that a lower limit of perhaps 15% may be needed; but likewise in this case the lower limit depends upon the permeability requirements, satisfactory compaction for stability of materials of glacial origin being usually secured without undue effort in ungraded materials with little or no clay content.

The writer recommends that, except for certain special conditions, permeability coefficients of the core material be restricted to less than 1 gal per sq ft per day. It seems that this restriction is a proper one for all dams to be used in storing water for water supply purposes, because the value of the water in such cases is usually high, and more money should be spent in obtaining a tighter structure than where the value of the water lost through the structure might not be as large. In earth dams built for hydro-electric developments the value of the water lost is usually not as great as in water supply dams; and in many cases it is economical and safe to construct impervious portions of the embankment from materials easily available, although they may not be as water-tight, as demanded by the foregoing requirements. In such cases the permeability coefficient could exceed 2 gal or even 3 gal per sq ft per day; and the stability of the structure, if proper drainage of the down-stream section of the dam were provided, would not be endangered.

The author's proposal to use the mechanical analysis curves in choosing the limits for suitable materials and, arbitrarily, to establish such limits for graded and ungraded materials, is a good one. However, in the curve limits suggested, the parts representing these limits are not useful above the 40 or 50% size, and the most important parts are those between the 0 and 30% size.

Fig. 16 shows the Kendorco[26] classification in reference to the limits established by the author. In this classification, the words, "uniform" and

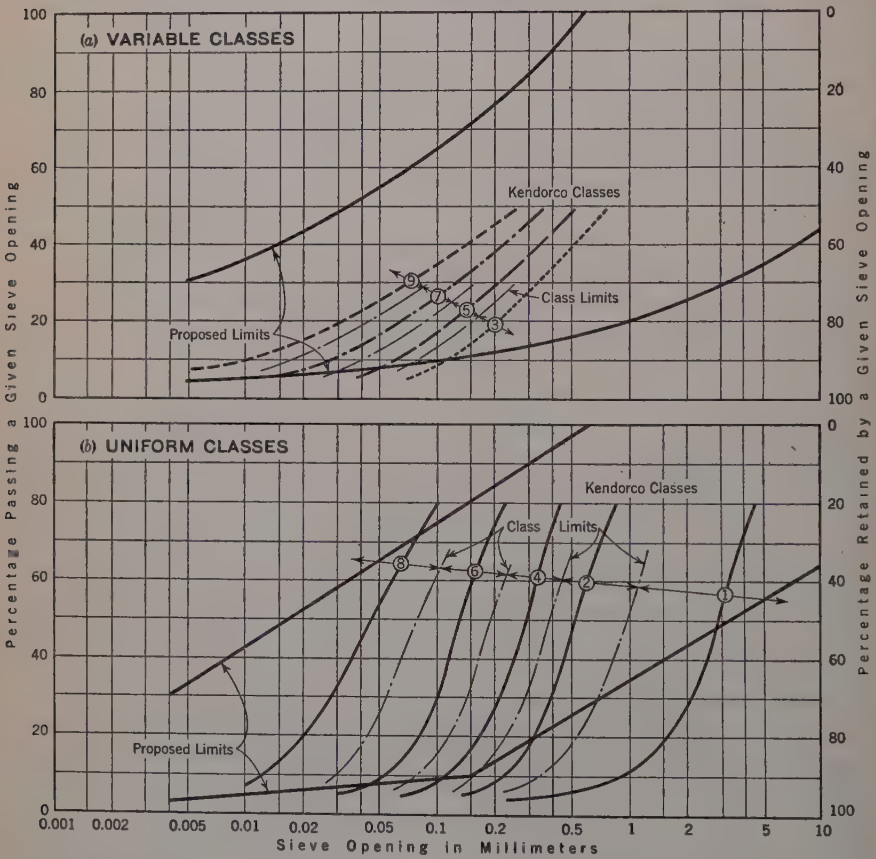

FIG. 16.—IMPERVIOUS SECTIONS OF ROLLED-FILL DAMS: COMPARISON OF PROPOSED LIMITS IN MECHANICAL ANALYSIS WITH GRADED AND UNGRADED MATERIALS OF GLACIAL ORIGIN.

"variable", are used in the same manner that the author uses "ungraded" and "graded." The finer of Classes 6 and 7 and all of Classes 8 and 9 represent materials suitable for impervious sections of the rolled-fill dams in accordance with the author's limitations. Classes 2, 3, 4, and 5, and possibly the coarser of Classes 6 and 7, represent materials that are probably unsuit-

[26] *Proceedings*, Am. Soc. C. E., March, 1936, pp. 306–307.

able, although the curves for them fall outside the prescribed limits only at the smaller sizes. Thus, it seems that the important part of the curve for material of glacial origin is in the vicinity of the 5 to 10% sizes only, and the location of the limits elsewhere is immaterial. Although the limits chosen by the author will be satisfactory in nearly all cases, they must not be used too rigidly as it is very probable that materials perfectly suitable for the purposes intended will be found, which, in some individual cases, will have mechanical analysis curves that will lie outside of one of the limits shown for a short distance.

JOHN E. FIELD,[27] M. AM. SOC. C. E. (by letter).[27a]—Engineers will welcome a technical treatment of the selection of materials for the more impervious parts of an earth dam, as a guide to design and construction of hydraulic and rolled-fill structures, and in masonry and rock-fill dams, where the necessity of blanketing portions of the reservoir basin sometimes arises, particularly in making repairs. Constructing engineers and engineer inspectors will benefit greatly from a knowledge of the theory on which the ideal mixture is based and toward which they strive.

There is much in the paper "between the lines" touching fundamentals, the most important of which, perhaps, is found in the sentence (see heading "Requirements for Suitability"), "the principal requirements for suitability of materials for the impervious part of an earth-fill dam are * * *." The word, "part", implies that a part only of the dam section need be impervious and that the portions below the "impervious" diaphragms may be, or should be, less impervious.

In the case shown in Fig. 17, the cause of the high and almost horizontal line of saturation was due to the uniformity of the material and compaction,

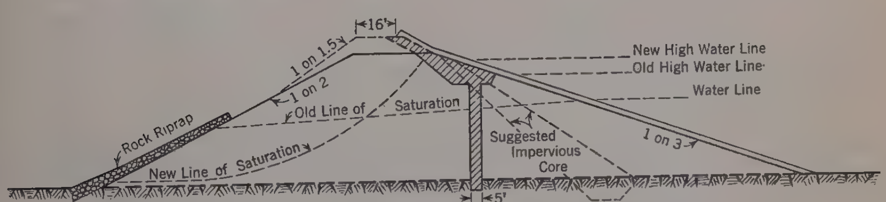

FIG. 17

and to its being too impervious in the lower or down-stream section. Of great importance also is the thought throughout the paper that a mixture of gravel and clay is a superior core material, for although this truth is well known, it is too seldom incorporated in plans and specifications.

Of the five requirements listed as "principal", the first and fifth might well be discarded in a theoretical discussion of the suitability of materials. As for Requirement (1), the danger of saturation and sloughing should be prevented by proper drainage and by rigidly confining the impervious core. Cost (see Requirement (5)) should not be considered in arriving at an ideal;

[27] Cons. Engr., Denver, Colo.

[27a] Received by the Secretary December 5, 1936

it should be considered only in the practical application to a specific problem after the ideal, toward which it is desirable to strive, has been fixed.

The ideal mixture, local materials considered, and the methods of attaining it, has use in many structures other than dams and where the quantities involved are small, although unit costs may be high as, for example, back-fill around culverts and siphons under large canals, the lining of canals passing over cavernous limestone or through very porous sections, and the back-filling of trenches in abutments and cut-off trenches of earth-fill dams.

The use of a nearly ideal mixture at considerable unit cost is shown in Fig. 17, a case in which the writer repaired an earth-fill dam. The dam, then about ten years old, had threatened to slough on the lower slope whenever the water rose to within 10 ft of the high-water line, and heavy riprap had been placed on the lower slope to enable the owner to use one-half the reservoir's capacity. The material in the fill was almost wholly of disintegrated granite containing some loam, clay, mica, and sand. It is unnecessary to give the details of the work, except to state that the mixture was 33% clay, with a small mixture of sand and loam, and 67% of the disintegrated granite excavated from the dam. The moisture content was about 10%, giving a stiff product easily handled and tamped; the width of the vertical trench was only 5 ft.

The repairs were successful in stopping all tendency to slough, the lower slope dried up and after five years' use no weakness has appeared. The cost was amply repaid by the gain in capacity above the old, theoretical, high-water line, the increased depth being about 7 ft. Notable are the very thin "impervious" vertical diaphragm, the more porous material back of it, and the change in the line of saturation.

With impermeability secured in a diaphragm, it is not always required that it have hardness, permanence, or stability. In the Necaxa Dam, and in many others of the hydraulic-fill type, the cores are probably of a consistency that would permit a lead weight to sink through their entire depth, but such a condition requires that the fluid core be firmly confined.

In all dams, weight is required to oppose the horizontal water pressures, and a high line of saturation and upward pressures decrease the effective weight. An impervious diaphragm and adequate drainage below it are effective and practical in lowering the line of saturation, and with these provided, the compaction of the body of the dam merely adds some weight and a more rigid support to the core, but adds nothing to its impermeability.

Therefore, it appears that a light diaphragm, made as impervious as possible with the ideal mixtures of sand, gravel, and clay, is more to be desired than a thicker section with an approximation to the ideal. Mechanical mixers of large capacity can be built and, in the writer's opinion, will be used in the not distant future to furnish the ideal material for the diaphragm, and at that time the formula and practice which the paper seeks to develop, will be used.

In his "Synopsis", the author mentions "the traditional manner of determining suitability of material" and intimates its inferiority to scientific

methods; but the use of scientific methods and the blind adoption of methods and formulas are equally dangerous, particularly if the cost is considered.

The late James Dix Schuyler, M. Am. Soc. C. E., states[28] that he had seen more mistakes in the ordinary earth-fill dam than in all other engineering structures coming under his observation. In this statement the writer heartily concurs, but wishes to add that the most numerous and serious mistakes he has observed in earth dams have been made by engineers of high technical attainments, particularly in the matter of cost, where the engineer failed to appreciate and take advantage of local conditions and materials, but adhered to theoretical formulas.

There is one desirable quality in fills of mixed clay and gravel not mentioned by the author, namely, that shrinkage cracks are lessened or obviated. In flood-control reservoirs, particularly in the arid regions, the dams may stand for years, become thoroughly dried out, and shrinkage cracks become a menace when floods fill the reservoir.

Referring to the use of tools and clogging (see heading "Workability"), the writer feels that the disk harrow is a valuable tool in securing uniformity of mixture and of moisture; that the mere stirring of the fill material gives opportunity for the smaller particles to find and fill the voids much more certainly than even the sheepsfoot roller; and that thorough mixing by the harrow obviates the danger of clogging the rollers and other tools.

Movement seems to be essential to thorough compacting and the moisture content should give sufficient fluidity to the material for it to move under the roller. A slight wave just ahead of the wheels of heavily loaded trucks, or a shallow rut pressed into the surface of the fill, indicates that the material is sufficiently fluid, whereas deep ruts indicate too much moisture and no ruts at all indicate too little.

Insolubility.—The writer has found as high as 24% of soluble matter in soils and in the case of one dam that failed he found, by the use of distilled water as a solvent, 8%, and by the addition of 1% of hydrochloric acid, 15%, of soluble matter. In every case, especially in the arid regions, solubility should be one of the first considerations and a knowledge of the geologic source of the material should be ascertained.

Neither clay nor adobe is essential to impermeability. Glacial flour or the slimes from stamp-mills are quite as impermeable and are of greater stability and higher frictional resistance. A search for, and recognition of, these materials and their use should be kept in mind in all glacial-deposit and mining regions.

In practice, the engineer first determines the ideal mixture of clay, gravel, and water. In construction, the percentage varies from time to time in spite of the most careful supervision and willing co-operation, and the author (see heading "Stability: Grading") advances the same idea: "In earth-stabilization practice * * * natural materials must be used, which conform as nearly as possible to an ideal grading."

Through "traditional" methods, the engineer-inspector and his assistants must be able to recognize material variations instantly and to apply the cor-

[28] "Earth Dams", by James Dix Schuyler.

rective measures at once, without waiting, for sampling and the results of laboratory tests; and eternal vigilance, and immediate corrective action is the price that must be paid for good work and proper progress. For quick inspection, the writer has found the use of the compression needle valuable, especially in impressing the dump boss and foreman with the idea that a covered-up fault would certainly be detected.

In the paragraph that introduces Table 2 is found mention of specifications "requiring exclusion of adobe soils." What is locally known as "adobe soil" varies widely in different localities, and a definition covering all varieties would probably be "a soil from which sun-dried brick can be made." It often contains considerable loam (another term extremely indefinite), and sand. In specifications, the writer would probably state that "adobe mixed with sand and gravel shall be used in the impervious core section", and add that "shales and indurated materials, although partly decomposed, shall not be used."

William P. Creager,[29] M. Am. Soc. C. E. (by letter).[29a]—In the writer's opinion, the author has done very well with the limited available data. He has advanced a step farther toward the establishment of definite limits beyond which it would be inadvisable to go in the use of soils for rolled dam construction; but, as he states, his hope is that his criteria will bring forth additional data from which may be evolved a basis for preliminary selection.

It is significant that his hope does not extend beyond criteria for a "preliminary" selection of materials. In this, he has shown himself conversant with the fact that many characteristics of soils, such as grain shape and grain composition, are not indicated by mechanical analysis but that they have much influence on the behavior of the material. Thus, at best, only a rough approximation of the essential characteristics of materials can be obtained from mechanical analyses.

His comparisons of mechanical analysis curves with stability are especially pertinent. Recognizing the well-known fact that the greatest density and, hence, greatest stability, results from a well graded mixture, he has shown in Fig. 1 the shape which curves of well and poorly graded mixtures assume on the usual chart of mechanical analysis. Fig. 1, together with his comparison of mechanical analysis with compaction curves, Figs. 3 and 4, brings out vividly that the best materials not only must be well graded between fixed limits, but must have a wide range of sizes, those lacking in either coarse or fine particles being less suitable.

Thus, in Fig. 4, the "fine sand" (Sample *L*1-1) has no very fine particles and hence is lacking in cohesion. Therefore, it possesses insufficient compressibility and squeezing properties to expel the entrained air effectively. It has low density and many air voids. Next above in Fig. 4 is the "sandy gravel" (Sample *L*15-*S*10), which contains a small percentage of fines. This is a fairly well graded mixture and hence has high density. However, it has

[29] Cons. Engr., Buffalo, N. Y.

[29a] Received by the Secretary, December 16, 1936.

insufficient fines for the proper compressibility and squeezing effect to expel a large part of its entrained air.

Closest to the curve of zero air voids are the loams, shown by full lines in Fig. 4. These loams are the ideal materials containing between 16% and 33% of clay accompanied by abundant coarse particles. They have the fewest air voids.

Finally, the silty clays and clays, Samples *L*15*CG*, *L*11-1, and *L*-8*A*, have a large percentage of fines, providing excellent squeezing properties; but they possess such low permeability that little if any draining of water and air can take place. Hence, they are of low density and have many air voids. It is difficult to account for the fact that Sample *L*11-1 shows up better than Sample *L*-8*A*. This is one of the inconsistencies that creep into the best of tests.

The writer feels that Fig. 6, defining proposed limits for graded materials is a great help in preliminary studies, provided: (1) That its use is tempered with experience and judgment; (2) that it is remembered that some materials outside these limits might be used; and (3) that some materials within such limits may be entirely unsuitable. Sample 42, for instance, lies wholly within the limiting curves, yet it has a compacted porosity of as much as 44.4%, and is quite permeable. This material is eliminated under the author's somewhat indefinite specification: "* * * it is not sufficient that the mechanical analysis curve should lie between the respective coarse and fine limiting curves; it must also have a shape corresponding to, or approaching, that of the limiting curves."

In order to assist in a graphical indication of this specification and more nearly define reasonable requirements, the writer suggests the possibility of using an additional curve, *AB*, Fig. 18, which would be the fine limiting curve for all materials having less than 15% clay content; that is, the curve for material having less than 15% clay content must not cross Curve *AB*. Thus, Sample 42, having less than 15% clay, lies wholly within the author's limiting lines, but it crosses the fine limiting line, *AB*, and, therefore, is undesirable, whereas Sample *L*15-*S*9 is on the order between good and bad material.

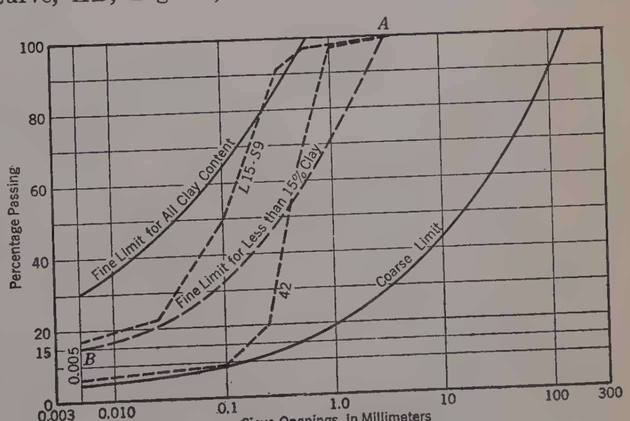

FIG. 18

It is not claimed that this additional curve will supplant judgment, but it is just one more step to assist in visualizing requirements. The writer's sug-

gestion is not confined to one additional curve. Other fine limiting curves might be used for materials having less than, say, 22% and 8% clay content.

In conclusion, the writer can see no practical need for Fig. 7 showing proposed limits for ungraded materials. Comparing the coarse limiting curves of Figs. 6 and 7 for graded and ungraded materials, any material having a curve which lies outside the ungraded limiting curve, but inside the graded limiting curve, automatically departs from the ungraded class, and Fig. 6 would apply. Referring to the corresponding fine limiting curves for graded and ungraded materials, it is the writer's belief that the difference between these curves is far less than is justified by present knowledge of the relation between suitability and mechanical analysis.

JOSEPH JACOBS,[30] M. AM. SOC. C. E. (by letter).[30a]—(1) The demand and need for a material expansion of water storage facilities throughout the United States, the economic importance of safety for the heavy investments involved in potential developments, and the fact that much of the required new storage will be effected by means of earth dams, renders this paper of timely interest.

(2) The earth dam is the oldest type known to Man. Where truly adapted to the site it is probably the best and most permanent type and, in fact, there are now existent more dams of that type than of all others combined. Despite these facts there is yet much to be learned concerning earth dam construction: How to select and effect the most economic mixture of available materials, the best degree and best methods of compaction, the stability of section, economic slopes, core-walls, and the best location and depth of cut-off walls, the density and permeability of various mixtures of earth materials, etc. It is mainly within the past decade that intensive study has been given to the problems of soil mechanics, the formulation of their basic principles, and the application of these principles to practical engineering. The entire realm of earth dam construction is still a fair field for research and this research should include not only detailed laboratory investigations, but also an intensive study of existing dams in respect of the aforementioned elements.

(3) In this connection it is of interest to contrast the densities that have been attained in compacted earth mixtures with those of concrete mixtures. In compiling certain data from engineering literature on earth dams, the writer finds that of about thirty actual tests of earth mixtures, for which mechanical analyses were given, the dry weight of the mixtures, after compaction, ranged from 96.7 to 132.5 lb, and averaged 115.4 lb per cu ft, or only 67.8% of the like weight of the constituent material. He also estimated, for each of these thirty tests, the theoretical maximum density that would obtain if all particles of the mix were placed so as to effect maximum density—that is, if the voids in each size of material were filled successively with smaller sized materials as far as these were available. This estimate indicated that the densities of the actual mixtures ranged from 71 to 96%,

[30] Cons. Engr., Seattle, Wash.

[30a] Received by the Secretary January 4, 1937.

and averaged 82%, of their respective theoretical maximum densities. These latter, depending on the particular proportions of the mix, ranged from 120 to 165 lb, and averaged 141 lb per cu ft, or about 83% of the weight of an equal solid of the constituent material.

(4) The foregoing results compare rather unfavorably with those obtained in concrete mixtures. In good, modern concrete, densities of from 155 to 157 lb per cu ft are readily obtainable if desired. Of this weight, about 6% is water, thus indicating, for the mixture, a dry weight of about 147 lb per cu ft, which is to be compared with the average of about 115 lb per cu ft for compacted earth embankments. Other comparative figures are: A voids content of only 13.5% for the concrete mixture as against 32.2% for the earth mixture; and a 6% moisture content for the former as against two to three times that proportion for the latter. It should be noted, too, that this modern concrete is compacted not by rolling or tamping processes but only by vibration.

(5) It may be contended that an earth embankment and mass concrete are two entirely different things and that they are, therefore, not directly comparable. However, in the initial condition of the mix, and in respect of density, a comparison may not be unfair for they are both composed of exactly the same types of materials, namely, earth and rock, the cement content of the one merely replacing a portion of the silt and clay content of the other. They differ only in their relative degrees of synthetic proportioning and mixing and in method of compaction, with a final result that one attains a density which on the average is one-fourth greater than the other. The contrast is striking and, at least, suggests the query as to whether engineers are now securing the maximum economic densities in earth mixtures and as to what may be the economic limits of proportioning, mixing, and compacting mixtures for earth dam construction. The legitimacy of this query will be appreciated when it is realized how materially seepage is reduced by an increase in the density of the earth mixture.

(6) Of the many considerations to be kept in mind in the selection of borrow-pit materials for earth dam construction, and in determining their proper mix proportions, particularly for the impervious part of the dam, it is believed that the following are among the more important:

(*a*) That the material shall be of sufficient weight, and shall be sufficiently graded as to particle sizes, to ensure a high density upon compaction, and thus to afford stability to the dam structure without imposing the necessity of extremely flat embankment slopes. Safe dams can be built of relatively light materials, but the heavier material is a desideratum in order that the expense involved in providing the extremely flat slopes, otherwise required, may be avoided.

(*b*) That the material shall afford a sufficient gradation of particle sizes, and particularly that there be a sufficient proportion of fines, to ensure against percolation velocities that might cause such erosion and piping within the body of the dam as to threaten its ultimate failure.

(*c*) That the material shall afford a sufficient gradation of particle sizes, the same as Item (*b*), but to ensure against a seepage volume deemed pro-

hibitive from considerations of economic water value alone, independent of possible physical damage to the dam by reason of such seepage.

(7) Assuming a satisfactory foundation, the stability of the dam proper, against imposed loads and attacking forces, depends mainly upon density of the embankment material in place; upon the position and gradient of the plane of saturation within the body of the dam and the volume of dam submergence below that plane; upon frictional resistance to displacement; and upon the cross-sectional form of the dam. The desired safety factor for stability, and the slopes and cross-section required therefor, are determinable from the other factors named. Most failures of earth dams have resulted either from their being overtopped by water, due to a lack of adequate spillway capacity, or to super-saturation followed by sloughing and progressive undermining toward the center from the outer slope or down-stream toe. There are few if any cases of earth dam failures that have been attributed directly to sliding. Such a failure, however, is conceivable. Under a condition of maximum loading as to water level above the dam, when maximum saturation obtains, the coefficient of friction of the embankment material is at a minimum, and, at such times, unless the base of the dam is adequately broad, a sliding movement would be possible as an incipient disturbance—and if sliding once began, disruption, subsidence, overtopping, and final failure would be likely to follow.

(8) Unless the embankment material is of reasonably good weight and grading, and well compacted, so that the plane of saturation is thereby well depressed toward the base of the dam, a condition obtains that makes for high porosity, high permeability, a high plane of saturation, a lowered resistance to movement under maximum load, and the necessity for flatter slopes and a broader base than otherwise would be required. This is due not only to the

TABLE 10.—Ratio, $\frac{\text{Base Width}}{\text{Height of Dam}}$, for Stability Against Sliding

Weight of embankment material in place after compaction and under full water load, in pounds	Height of Dam, in Feet:							
	50		100		200		300	
	Coefficient of Friction							
	0.25	0.50	0.25	0.50	0.25	0.50	0.25	0.50
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
120	4.9	2.4	5.1	2.5	5.2	2.6	5.2	2.6
100	6.3	3.0	6.5	3.2	6.6	3.3	6.6	3.3
80	8.7	4.2	9.0	4.4	9.0	4.5	9.2	4.5

lesser basic density of the embankment and the greater volume of submerged material that loses weight by buoyancy, but also to the fact that a lesser coefficient of friction obtains under these conditions. This is illustrated in Table 10, which shows the required ratios, $\frac{\text{base width}}{\text{height of dam}}$, for the single consideration of stability against sliding. Other considerations may, and in many cases will, require base widths different from those indicated.

(9) The assumptions upon which the data in Table 10 were developed, were as follows: A stability factor of two against sliding; dam heights ranging from 50 to 300 ft; a top width of 25 ft; a down-stream slope (horizontal ÷ vertical) two-thirds that of the up-stream slope; a mean weight of embankment material, above and below the plane of saturation, ranging from 80 to 120 lb per cu ft; and a coefficient of friction ranging from 0.25 to 0.50. Although greater unit cohesion and shear strengths than are indicated by these friction coefficient values are quite possible for dense, well compacted materials, it is believed that for conservative design, values of less than 0.50, and probably not to exceed 0.25, should generally be assumed. It will be noted from Table 10 how materially the required base width increases with a decrease in value of the coefficient of friction and a decrease in the density of the embankment material. Regardless of the showing of Table 10, however, or of stability computations in general, the writer would not favor earth dam slopes steeper than 1 on 2, nor base width ratios of less than 4.2 to 1, except possibly for very low dams.

(10) In this connection it is of interest to note the early British practice in India where so many excellent earth dams have been built. In 1910, Mr. A. Hill, Chief Engineer, Irrigation, presented data concerning some of these dams in a report entitled "Saturation of High Embankments of Storage Tanks in the Bombay Presidency." Although this report (which dealt with twelve dams from 31 to 81 ft in height) unfortunately contained no detailed data as to the density and mechanical analyses of the embankment materials, it did indicate that they were of average good quality, and it also contained detailed data as to embankment slopes and planes of saturation. From this report, the writer deduced the information contained in Table 11. There were some short sections of embankment slope much steeper and some much flatter than the averages shown in Table 11. Some were as steep as 1 on 1 (near the top) and some as flat as 1 on 14 (near the bottom). Furthermore, there were some short sections of saturation slopes as steep as 1 on 0.5 and as flat as 1 on 60.

TABLE 11.—DATA CONCERNING TWELVE EARTH DAMS IN INDIA

Item No.	Ratios	Averages for Individual Dams		Mean for all dams
		Maximum	Minimum	
1	Base width / Height of dam	6.7	4.9	5.70
2	Up-stream slope	3.6	2.5	3.04
3	Down-stream slope	3.9	2.1	2.52
4	Plane of saturation	7.6	1.9	3.6

(11) The author suggests that the qualitative desirability of materials for earth dam construction, both as to permeability and as to density, after compaction, may be judged by determining, from mechanical analyses of the materials, their value of n in the Talbot formula (Equation (1)). As to permeability, the writer questions the validity of this criterion, but believes that it is a fairly dependable index of potential density. Assuming a maxi-

mum particle size of 6 in. (152.4 mm), and a uniform value of n throughout for any particular mix, as the Talbot formula provides, the writer determined the theoretical proportions of the various particle sizes for that mix. He also considered a series of arbitrary mixes wherein the value of n was variable and for these the mean value of n was determined for each particular mix. Probably few natural mixes have a uniform value of n throughout. If it were deemed important (and the writer does not so regard it), a uniform value of n could be attained by extensive artificial proportioning; but such extensive proportioning might be quite infeasible economically. Generally, the engineer must utilize, to the best advantage possible, the materials immediately at hand.

(12) For each of the mixes studied a theoretical maximum density was estimated, this being on the assumption that the various sized particles all found that proper place in the mix which would effect maximum density—a condition not fully attainable in practice. As already stated in Paragraph (3), of about thirty or more definite tests of mixtures and densities, for which data were available, the actual densities ranged from 71 to 96%, and averaged 82% of the theoretical maximum density. An 80% average would seem to be quite readily attainable. The theoretical mix of a uniform value of n, and the arbitrary mix of an equal mean value of n, are found to be not entirely accordant as to resultant densities, but they seem to be sufficiently in agreement to permit adjustment to a common density curve There is less accord as to permeability.

(13) From the computed n-values and densities for various hypothetical mixes, and from the recorded tests of numerous actual mixes, a graph (see Fig. 19), was developed showing density variation with variation in value

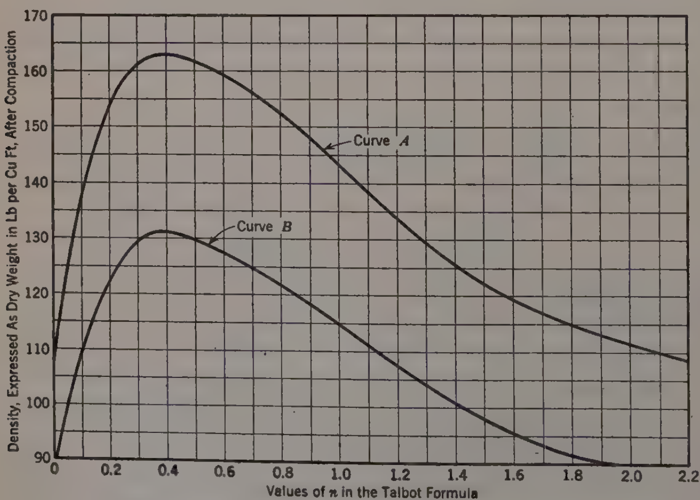

FIG. 19.—ESTIMATED DENSITIES OF EARTH MIXTURES FOR VARYING VALUES OF n IN THE TALBOT FORMULA (EQUATION (1)): $P = \left(\frac{d}{D}\right)^n$.

of n. Curve A, Fig. 19, is for the theoretical maximum densities, assuming that all particles are so disposed in the mix as to effect maximum densities; and Curve B is for practically attainable densities taken as 80% of the theoretical maximum. The maximum particle size, D, is assumed to be 6 in. (= 152.4 mm). These curves are fairly close compromise adjustments to the platted actual computations. It will be noted from Curve B that maximum densities obtain for n-values ranging from 0.30 to 0.50, the actual maximum appearing to result for $n = 0.38$. It will be noted, too, that for densities as low as 115 lb per cu ft (which, as already indicated, is about the average now attained in actual practice) the value of n may range from 0.14 to 1.0. Although these values, both of which apply to 115 lb per cu ft, may indicate mixtures acceptable as to density, the latter also indicates a mixture entirely prohibitive as to seepage volume. Therefore, n alone, unless it is rigidly restricted within narrow limits, will not suffice as a criterion of all requirements for a good earth mixture for dam construction.

(14) The author states that a seepage loss of 0.1 gal per day per sq ft and, under certain conditions, in excess of 1.0 gal per day per sq ft, might be permissible. Ignoring the question of possible piping, and considering only the economic value of the water loss, it is not a simple matter to determine permissible seepage. It depends upon many factors: Primarily, upon the actual value of water in the particular locality and for the particular project involved, but also upon the relation of the monetary value of the annual seepage loss to the total annual cost of the project; upon its comparison with the extra cost of alternate methods of water securement that might avoid the seepage; upon whether or not the seepage water can again be utilized farther down stream, etc. Water values may range from \$500 to \$50 000 per cu ft per sec and storage costs from \$2 to \$200 per acre-ft. High seepage loss is to be avoided if possible, of course, and other considerations may make it entirely untenable, but from the sole consideration of the economic value of the water, the writer believes that seepage losses several hundred-fold greater than that indicated by the author (perhaps 500 to 1 000 times as great) may be permissible. He is confident, too, that there are many entirely stable earth dams with seepage losses as great as 5 gal per day per sq ft.

(15) Permeability depends mainly upon the porosity and "effective size" of the dam material in place. For mixtures of regular grading, according to the Talbot formula, there is a definite "effective size" for every value of n. From known or assumed compaction densities, as, for example, those shown in Fig. 19, porosity is readily determined. From these factors, an assumed water gradient of 1, and an assumed water temperature of 60° F, the writer determined, on the basis of the Slichter formula, the permeability of various mixes for varying values of n. These indicated that to keep within the maximum limits of permissible seepage as discussed in Paragraph (14) the value of n should not exceed 0.4 and the "effective size" should not exceed 0.5 mm. It also indicated, for the seepage limit designated by the author, an n-value of 0.26 and an "effective size" of only 0.025 mm.

(16) Such computations, however, can at best be accepted only as rough guides. In the first place, mixtures of irregular grading, but of the same

mean value of n, give "effective sizes" and permeabilities that differ somewhat (and, in some cases, appreciably), from those of the regular grading determined by the Talbot formula. In the second place there is much uncertainty as to the dependable accuracy of any of the present empirical formulas for ground-water movement as applied to the type of earth mixtures considered herein. It should go without saying that in actual dam construction, empirical formulas, to which have been applied the mechanical analysis factors of the earth mixture, should not be the main reliance for permeability determinations. There should be a comprehensive program of permeability tests of the actual materials, mixed and compacted to a condition as nearly that which will obtain in the final dam, as is possible.

(17) Seepage velocities that may cause piping should be avoided, of course. Generally, piping is a slow, continuing process that leaches out particles, progressively from fine to coarse, and which, unless halted, means ultimate structural collapse. The earth mixture, therefore, should be such as will permit no seepage velocities capable of dislodging and moving out the smallest sized particles which the dam material may contain in measurable quantity. A certain percentage of fines is an essential of good earth dam construction and some of these fines may be clay particles as small as 0.0005 mm in diameter. The writer estimates that for a safety factor of two against movement of such a particle, considering weight only and ignoring friction, the limiting water-jet velocity would be 0.00545 ft per sec, and that is the desirable limit of seepage velocity if practically obtainable. However, because there is a material friction and surface tension between particles, and because the 1 on 1 gradient upon which permeability factors are based is greater than obtains in a well built dam, an appreciably greater velocity than that stated would be permissible as far as piping is concerned. The seepage volume factor, however, intrudes itself and prevents any very material increase of that velocity. For a seepage velocity of 0.00545 ft per sec, through regularly graded Talbot mixes, the writer finds a limiting n-value of about 0.4 and a limiting "effective size" of about 0.5 mm.

(18) Seepage loss is so important a factor in connection with storage problems, and in relation to dam design and construction, that, unquestionably, there is an urgent need for further research concerning the laws governing ground-water movement. Every engineer who has had contact with the problem is aware of the glaring inconsistencies that obtain in the recorded data relating to soil permeability and to ground-water movement in general. These inconsistencies are revealed not only in the comparison of actual tests with estimates based on empirical formulas, but also in a comparison of the actual tests themselves, independent of any estimates by formula. Numerous instances can be cited of great disparities in actual tests of seepage through materials practically identical as to density, porosity, and "effective size"—and even greater disparities between actual tests and formula estimates. From a number of actual permeability tests of materials for which the writer computed discharge by one of the generally accepted empirical formulas, he found disparities as shown in Table 12. The degree of accuracy of the actual

tests is not known but, presumably, they were made with care and are reasonably accurate. In any event, it is clear that either the tests or the formula as applied to graded earth mixtures is grossly in error, and it is confidently believed that it is the latter.

TABLE 12.—PERMEABILITY COMPARISONS, SHOWING INCONSISTENCIES

Case	Mechanical Analyses of Materials				Ratio: Actual test discharge / Formula discharge
	Dry weight per cubic foot, in pounds	Percentage of voids	Effective size, in millimeters	Uniformity coefficient	
1	96.7	45.0	0.250	4.0	0.96
2	119.4	34.0	0.559	9.2	2.14
3	109.3	38.0	0.066	3.0	0.0405
4	119.4	34.0	0.760	14.7	0.0149
5	117.5	34.0	0.102	18.1	0.414
6	127.0	30.0	0.100	95.0	0.000284
7	123.0	29.3	0.00085	2355.	14.35
8	113.4	31.9	0.730	5.35	2.70

(19) Most of the permeability formulas have been devised for a more or less uniform and a relatively fine material and were not intended, primarily, for such graded mixtures as are customary for earth dams. They seem, particularly, to lack applicability to coarse media, giving impossibly high velocities. They may be serviceable indicators as to the relative permeabilities of different mixtures, but should be interpreted with great caution as to absolute permeability. It may be that in the tests from which these formulas have been developed adequate cognizance was not given to the factors of entry and exit head losses; there may be error in the assumption that velocity varies as the first power of the head (and the writer believes there is error in that assumption as applied to coarse media); or there may be other factors in the problem that have not yet been discovered. Wherever the difficulty may lie, however, it is evident from the great disparities in results, to which reference has already been made, that further research is badly needed. It is the writer's judgment that this research should contemplate, in addition to laboratory tests, an intensive investigation of existing dams as to density, seepage, planes of saturation, mechanical analyses of the materials of the dam, etc. Such an investigation would involve considerable expense, but it is the safest procedure for the development of dependable formulas for the permeability of earth dams.

(20) As final comment and, in part, as résumé of what has preceded, the writer submits the following:

(*a*) To satisfy best all requirements, that is, to secure an acceptable density and to keep within permissible limits of seepage velocity and seepage volume, the earth mixtures should preferably be such as to have an *n*-value within the range, 0.25 to 0.40, and an "effective size" not to exceed 0.5 mm. On account of the better workability of the material, due to the larger percentage of fines, the lesser values of *n* are the more desirable although it means, also, a slight sacrifice of density.

(*b*) It is permissible, and often necessary, to exceed the limits indicated in Item (*a*) but the embankment design, of course, must be adapted thereto. It may mean flatter slopes, a greater provision for drainage, the need for a core-wall, or of additional cut-off walls, etc., depending upon the degree of departure from the ideal material. If, for example, the value of n exceeds 1, the dry weight density will fall below 115 lb per cu ft and a base width greater than five times the height of dam will be required for assured stability.

(*c*) For the limits designated in Item (*a*), the percentage of fines, namely, material of less than 0.05 mm size, will generally not exceed 15% and may be somewhat less. With the present better knowledge of mixing and compaction methods, these relatively small percentages of fines are not only permissible, but they actually give a final density exceeding that of mixtures containing larger percentages of fines. It contrasts rather sharply with the one-quarter to one-third, or more, proportion of clay formerly deemed essential.

(*d*) To what extent economic considerations will permit increased expenditure to secure greater density of embankment materials by means of more thoroughy proportioning, mixing, and compacting than is now customary, depends mainly upon the number and proximity of available borrow-pits and upon their character in respect of the amount of additional processing that may be required. The fact that additional mixing may be rather expensive, and that the denser mixture requires more borrow-pit material for the same embankment volume, tends to limit the extent to which extra processing can be economically utilized. Generally, the available materials must be taken as they are, with no more processing than results from the excavation operations in the borrow-pit and from the spreading and harrowing or disking on the dam. The simultaneous use of borrow-pits of somewhat divergent composition may permit a nearer approach to the desired mixture, without undue synthetic proportioning, than would otherwise be possible

(*e*) An intensive research as to the permeability of earth dam embankments under varying conditions is urgently needed. It is a project that should be undertaken by the Federal Government through one of its regular technical agencies.

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DISCUSSIONS

STRUCTURAL APPLICATION OF STEEL AND LIGHT-WEIGHT ALLOYS A SYMPOSIUM

Discussion

BY MESSRS. ELMER K. TIMBY, WERNER LEHMAN, OTIS E. HOVEY, AND R. G. STURM

ELMER K. TIMBY,[125] ASSOC. M. AM. SOC. C. E. (by letter)[125a]—The types of tests which may confront the designer of structures, the test methods and materials which he may use, and the technique necessary for the correct design of the model and interpretation of experimental data are outlined clearly in Mr. Templin's excellent paper. It is the purpose of this discussion to show to what extent models are actually used by the profession in the advancement of structural design methods.

Every useful structure develops a definite stress system under a given load condition. The determination of this stress system is the concern of the designing engineer. A simple structure may be solved by the well known equations of equilibrium; hyperstatic structures require a greater number of equations than can be written from the condition of equilibrium, and the sources usually drawn from are the principle of least work and the theory of consistent elastic distortions. The formation of these equations frequently involves assumptions of important magnitude—assumptions that may be so far in error, if improperly made, as to vitiate completely the results of the solution. The model, on the other hand, automatically solves the problem, avoiding the use of equations and the assumptions on which they are based.

The summaries and illustrations that follow are an attempt to review briefly the results of past work in this field in the hope that those who have not had an intimate connection with the development of this line of attack

NOTE.—This Symposium was presented at the meeting of the Structural Division at Pittsburgh, Pa., October 14–15, 1936, and published in October, 1936, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: December, 1936, by Messrs. E. Mirabelli, R. W. Vose, Raymond H. Hobrock, William F. Clapp, J. C. Hunsaker, Horace C. Knerr, and F. T. Sisco; and January, 1937, by Messrs. J. Charles Rathbun and D. M. MacAlpine, Fred L. Plummer, C. F. Goodrich, G. K. Herzog, John H. Meursinge, P. C. Lang, Jr., and W. L. Warner.

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[125a] Received by the Secretary December 2, 1936.

may have a better conception of the true amount of work done and of the effectiveness of the method. The results presented have been assembled partly by circularizing engineers known to be interested in this work. The response to these inquiries was most gratifying, and it is hoped that the summary represents a typical cross-section of the opinions of the profession. It is pointed out that these examples by no means represent all the work which has been done. Many very important studies have been omitted because of lack of space.

One classification includes those models which have formed the basis of research projects sponsored by technical societies, universities, or industrial organizations for the purpose of promoting the progress of engineering. One of the most outstanding examples of this type of work was the extensive use of models by the Committee on Arch Dam Investigation.

The Stevenson Creek Test Dam, studied by this Committee, might be called a full-sized model. It was constructed at a particularly favorable location along Stevenson Creek, in California, where a deep reservoir having a very small capacity was possible. A large source of water, the flow of which could be controlled, was available up stream. The dam was 60 ft high, 2 ft thick at the top, and thickened along a curve on the down-stream side for the lower half of its height.

This dam was loaded and unloaded repeatedly and many observations were taken in an effort to determine its stress distribution. The results obtained have been published in several volumes by the Engineering Foundation[126]. Many organizations and individuals co-operated in making the tests and the results were of sufficient importance to warrant the Committee to make certain general recommendations helpful in designing, constructing, and testing future arch dams, and in testing existing dams to establish their safety, or to determine the feasibility of increasing the height or making other alterations.

The cost of conducting a test on such a large scale was necessarily very great and for this reason the Committee decided to investigate the same structure by use of a celluloid model. This was done at Princeton University under the direction of George E. Beggs, M. Am. Soc. C. E. The model was made of amber celluloid, placed in a concrete valley and loaded with mercury. The results obtained agreed so well with the tests on the full-sized model that future studies to determine the effect of increasing the height of the dam were completed by use of additional celluloid models at a cost much less than would have been required to raise the concrete dam.

It is interesting to note one point brought out by the model studies. Under certain conditions of loading, a part of the dam deflected up stream against a uniformly varying water load. After the model had demonstrated that this was true, a theory was promptly furnished which would lead to this result. Theory had previously erred in assumptions made; experiment corrected the error, and, consequently, improved the theory.

[126] Rept of the Committee on Arch Dam Investigation, Engineering Foundation, Vol. 1, 2, and 3.

The particular usefulness of the tests on the model of the Calderwood Dam[127], together with the tests of the structure itself, was that they emphasized the inadequacy of considering an arch dam as a simple series of arches and indicated definitely that some more complete method of analysis, such as the 'trial-load", method is necessary. They further emphasized that in any analysis the character and deformation of the foundation must be considered.

Supporting these observations relative to model studies of dams are the following conclusions which are quoted from a paper by J. L. Savage, M. Am. Soc. C. E., presented at the 1934 Annual Meeting of the Society for the Promotion of Engineering Education[128]:

"The careful testing of accurately constructed model dams furnishes a satisfactory and reliable basis for checking the action and safety of concrete arch, arch-gravity, and gravity type dams as well as a satisfactory means of verifying analytical methods used in designing such structures."

* * * * * * * * *

"Plaster-celite models of maximum sections of arch, arched-gravity, and gravity dams are useful in determining load deflections, general stress conditions, special stress conditions at corners, stresses around galleries, and so forth; but may give misleading stress curves in the foundation material due to the necessity of using rigid boundary conditions."

* * * * * * * * *

"The experimental analysis of dams offers an unusually fertile field for further research and for the instruction of students."

Models also have been extremely useful in the construction of huge dams, as well as of assistance in design. The use of models in connection with the construction of the Bonneville Dam (Oregon) will serve as a timely illustration. Of this project, J. C. Stevens, M. Am. Soc. C. E., has stated[129]:

"Hydraulic models of the Bonneville Dam have saved many times their cost and have substantially accelerated the construction schedule of the project. The experiments have been carried on in an outdoor hydraulic laboratory built especially for the purpose at Government Moorings, in Portland."

In describing a model type of baffle adopted to prevent scour and consisting essentially of blocks staggered in two rows, Mr. Stevens stated[130]:

"It was a surprise to find that the downstream sloping face of the baffle block was very effective in preventing scour, but once the fact was established a theory to account for it was discovered in the phenomena of turbulence and stray currents."

The coffer-dam design and installation were studied thoroughly by models.

The Special Committee of the Society on Concrete and Reinforced Concrete Arches made many model studies in connection with concrete arch bridges, on rigid abutments, on elastic piers, with and without high and low superstructure, and with and without expansion joints. A number of both concrete and celluloid models were used in these tests. The experimental

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[127] "Model of Calderwood Arch Dam", by A. V. Karpov and R. L. Templin, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E. Vol. 100 (1935), p. 185.

[128] A condensed version of this paper was published in *Engineering News-Record*, December 6, 1934, under the title, "Dam Stresses Studied by Slice Models."

[129] *Civil Engineering*, October, 1936, p. 674.

[130] *Loc. cit.*, p. 675.

work on the concrete arches was conducted at the University of Illinois under the supervision of W. M. Wilson, M. Am. Soc. C. E.[16] Celluloid model studies of some of the same arches were made at Ohio State University under the direction of Clyde T. Morris, M. Am. Soc. C. E., and at Princeton University, by Professor Beggs[131]. As a result of these and other tests conducted by it, the Committee has made definite recommendations for design practice.

Cases of the use of models to improve theory have been cited by Mr. Templin. It might be well to consider a case in which a model was used to verify a new theoretical approach. The geometric properties of the Williot diagram were used to express moments in terms of angular displacement and thus furnish a sufficient number of equations for the stress analysis of the tower of the Golden Gate Suspension Bridge. This was a large and unusual cellular structure combining set-backs with portal construction above the roadway. Below the roadway cross-bracing connected the two legs of the tower. It was considered desirable to verify the theoretical design by means of an independent study. Professor Beggs constructed a structural model and made strain and deflection measurements at many points. Then he computed the model, using the theory which was being applied in the design in the full-sized tower.

This model was fabricated from stainless steel by shot-welding to a scale of 1 to 56. Tensometers were used to measure strains; deflections were measured with dials; and rotations were measured by the use of mirrors and reflected light. Structurally, the model was a duplicate of the tower as regards area of members, distances to extreme fibers, and moments of inertia. However, the entire comparative theoretical work was based on a calculation of the model. The results of the experiment were in excellent agreement with the results of the theoretical calculations. The model was used in this case to verify a method. There could be no question of lack of similarity since the identical structure was used in each of the two methods. The theoretical method can now be applied with confidence to structures of a similar nature.

In addition to the verification of a particular method this model represented a new type of model construction. Stainless steel is a material of exceptional quality and when fabricated properly, produces a structure in which there is no question as to the rigidity of joints. Some engineers may question the use of welding to represent structures which in practice are assembled by riveting. It is the general practice, however, for designers to consider the gross area and moment of inertia of either riveted or reinforced concrete members during the calculations of resultant forces, and to make allowance for net sections, or cracked sections, only when the point at which the unit stresses are to be evaluated has been reached. For this reason it is thought that the use of welding is entirely justified. Allowance for net sections may be made when interpreting the observed strains. Theoretical deflection calculations are also generally based upon gross sections and, therefore, need no correction when welding is used instead of riveting.

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[16] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 154.

[131] Final Report of the Special Committee on Concrete and Reinforced Concrete Arches, *Transactions*, Am. Soc. C. E. Vol. 100 (1935), p. 1427.

It seems entirely reasonable that the use of gross sections should give results close to the actual even when a structure is assembled with rivets. The rivet holes form such a small part of the structure that their total effect in any case would be slight. Furthermore, the irregularity of stress around a rivet hole argues against too great refinement when considering the over-all action of a structure using theories postulating uniform stress variation.

Another example of the use of models is in connection with suspension bridges. In the final report[132] on the Delaware River Bridge, Leon S. Moisseiff, M. Am. Soc. C. E., presented a treatise on suspension bridge theory. D. B. Steinman, M. Am. Soc. C. E., applied this theory to the Mt. Hope Suspension Bridge, in Rhode Island, and furnished rather complete theoretical calculations for this structure[133]. Since such complete data were available, Professor Beggs selected this structure as the prototype for a model study to determine whether a model of a suspension bridge would yield satisfactory design information.

The results of this study were in excellent agreement with the available theoretical data, and it was evident that a vast amount of information could be obtained from such a model in a relatively short time. Slight but consistent differences between experimental and theoretical values were explained by the fact that the theory assumed the application of uniform suspender loads, as a matter of convenience, whereas the model showed definitely (as was expected) a non-uniform distribution of load[134].

Although the model of the Mt. Hope Suspension Bridge was made and tested purely as a research problem, it did prove so practical that Professor Beggs later undertook preliminary studies of the San Francisco-Oakland Bay Bridge, using models of the type developed in the Mt. Hope studies. The models, three in number, followed preliminary designs of the full-sized structure and were made from the following data furnished by these designs: (1) Principal dimensions of the structure; (2) cross-sectional areas and elastic moduli of cables and suspenders; (3) moments of inertia of the stiffening trusses and the elastic modulus of their material; (4) elastic constants of the towers; (5) magnitudes of the various loads; (6) general proportions of certain parts, such as the tower saddles and cable anchorages; and (7) range of temperature conditions.

Information obtained from the tests included tower deflections, cable deflections (both lateral and vertical), truss deflections (both lateral and vertical), cable stresses, grade changes on the roadway, and lateral and vertical truss moments. Instrument readings were converted directly into values for the prototype by means of properly constructed charts based upon elastic constants, scale factors, and calibration factors. The results[135] were most useful, both from a research and from a design point of view. One of the models con-

[132] Final Report of the Board of Engineers on the Bridge Over the Delaware River Connecting Philadelphia, Pa., and Camden, N. J., Appendix D, p. 96.

[133] "Suspension Bridges", by D. B. Steinman, Second Edition.

[134] "Suspension Bridge Stresses Determined by Model", by George E. Beggs, M. Am. Soc. C. E., Elmer K. Timby, Assoc. M. Am. Soc. C. E., and Blair Birdsall, Jun. Am. Soc. C. E., *Engineering News-Record*, June 9, 1932.

[135] "Tests on Structural Models of Proposed San-Francisco-Oakland Suspension Bridge," by Messrs. Beggs, Davis, and Davis, Univ. of California Press, 1933.

formed very closely to the design finally adopted and was partly dismantled, during the construction of the prototype, and used to plan the sequence for erecting stiffening-truss sections without producing excessive tower deflections, and to determine the order of erecting the floor steel, and the order of placing concrete paving.

The model furnished a means of providing easier, quicker, cheaper, and probably more reliable information than could have been obtained analytically. Early measurements of tower deflection varied 10% or more from those measured on the prototype. The variation, however, was uniform and after the law of variation was established, proper allowance was made and subsequent behavior predicted with satisfactory accuracy. Construction operations were expedited to an extent that made the mechanical analysis more than self-supporting. No accurate costs of this test are available, but it is estimated that it was less than 0.1% of the contract. The tests were made jointly by the engineer and the contractor and were accepted by the engineer as a basis for the approval of the erection program.

Another interesting experiment which led from the Mt. Hope model was the model which was used during the erection of the George Washington Bridge across the Hudson River, at New York City. During the erection of this bridge some of the junior engineers in the field had witnessed a demonstration of the Mt. Hope model. Upon returning to the job, one of them, Mr. A. O. Bergholm, decided that it would be comparatively easy and extremely interesting to construct a model of the partly completed bridge and have a private check for his own use of the erection data furnished by the design office. Accordingly, at his own expense, he began the construction of a model in the basement of the building being occupied as the Field Office. His project was discovered, however, by the Resident Engineer who was so favorably impressed by the possibilities that he authorized the necessary expenditures for constructing the model.

In this model the cable and suspenders were made from piano wire, the dead weights from cloth bags and shot, and the towers and foundation from wood. Dials were utilized to measure saddle motions and a vertical scale was used to determine the position of each panel point. In addition to being used many times, the model saved its cost by solving one particular problem.

Steel for the roadway was delivered to the structure by barge. On one day the steel arriving on the barge was different from that called for on the erection schedule. The question arose as to whether the erection schedule could be altered safely and the erection of the steel which was on the barge permitted. The engineers went to the model, applied an amount of lead shot which corresponded to the steel in question and observed the deflections. The results indicated that the steel could be erected safely at that time and the erection proceeded without delay.

The rapid determination of an erection schedule is always extremely vital to the bridge fabricator. Years may have been spent by the promoters of a project before arriving at a decision to construct, in smoothing out legal difficulties, and in actually undertaking the contract; but, when the fabricator

comes "into the picture", time is at such a premium that the fabricator may be liable to a late penalty of as much as $1 000 per day on a large project. The fabricator, therefore, is anxious to use a reliable means of determining his erection schedule quickly because upon this hinges the sequence of his mill and equipment orders, and the profitable completion of his contract.

The use of models of suspension bridges by bridge companies for the determination of erection data is one of the notable examples of the practical application of models in the field of bridge construction. One company has constructed models of four bridges, in each case for the primary purpose of determining definite erection data for the trusses and roadway. The first such model used by this company was utilized in determining the erection schedule for the reconstruction of the Ambassador Bridge, at Detroit, Mich. This bridge has an 1 850-ft main span and unloaded side spans.

The model was ingeniously designed, of inexpensive material, in such a manner as to be quickly fabricated. Nominal $\frac{1}{8}$-in. bead chains, similar to pull-chains for electric lamps, were used for the cable. This was flexible, permitted easy location of the suspenders, and had a weight which gave a practical scale reduction factor. The saddles of the prototype were mounted on rollers during erection and, to simulate this freedom of motion in the model, the tower was replaced with a hinged tension member that could be made vertical by adjustment and comparison with a plumb-bob. Being vertical it exerted no horizontal component, and thus simulated the rollers. For that part of the span with suspenders only hanging from the cables, narrow strips of paper simulated the weight of the suspenders. Pieces of soft iron wire were used to represent the weight of the suspender plus the truss and floor system. One man spent two weeks in designing and erecting the model.

Information obtained from this model consisted of the shape of the cable under unsymmetrical or partial erection loads with particular reference to kinks at cable bands and at the edges of the saddles, vertical deflections of the cable, horizontal movements of the saddles, and the allowable or necessary sequence of closing the joints in the trusses. This information was determined for successive conditions of erection and for various numbers of travelers and the total time required to obtain all this information was one man's time for one week. As a result of the studies, the eight floor-beams at mid-span were suspended from the cable before the travelers left the towers. This pre-loading eliminated excessive cable kinks.

The results obtained from this simple model were entirely satisfactory for field use and were subsequently checked by available and well established, but nevertheless time-consuming, analytic methods. The agreement between experiment and theory was also entirely satisfactory.

The fact that the aforementioned model was practical and economical as well as rapid is best established by noting that the same company has since used three similar models in the determination of erection data.

A model was made for the Triborough Bridge in New York City. The engineer for this bridge had fixed definite limits for the tower deflections. Therefore, it was essential to have material arrive on the job in an order which would insure freedom from delays caused by revision of the erection

schedule, which might be necessary should the adopted plan of erection produce excessive tower deflections. Model studies furnished a theoretically ideal erection schedule and allowable variations therefrom. This schedule was used during the actual erection of the bridge and proved to be entirely satisfactory.

The Triborough Bridge was chosen as a test bridge for the comparison of erection data obtained from a model, from observation of the bridge during erection, and by calculation. Comparisons have been entirely satisfactory for erection purposes and the model has shown itself to be superior to computation even when the erection schedule had been definitely predetermined and, therefore, all trial calculations eliminated.

Since 1928 another bridge company has used two models to determine the dampening effect of various types of storm systems. Any one familiar with suspension bridge erection will know of the complicated nature of these systems on long-span bridges and will be able to appreciate the effectiveness of a model in studying their action. Other models have been made so that spinning equipment could be operated in miniature. The rapid strides made in the art of spinning parallel wire cables is ample proof that the models have been useful. The equipment used and the spinning speed attained during the spinning of the cables for the Golden Gate Bridge mark another milestone in suspension-bridge construction. By the use of triple spinning wheels and double tramways, twenty-four wires were placed with each trip of the wheels as compared with only four wires on the George Washington Bridge, the cables of which were erected only about eight years before. Models have played no small part in this rapid progress. All the models used by the company have either reduced the cost of the construction or the time required for design to such an extent as to be more than self-supporting. As a matter of interest, where records are available, these various model studies cost from 0.05% to a maximum of 2.5% of the cable contract.

Other models have been used by the company to study erection equipment for foot-bridges, cables during the erection of the roadway, and suspended cable-car transportation systems. All materials used have been inexpensive, and the models have been fabricated in an exceedingly practical manner. The erection procedures for the cables of the George Washington Bridge, the "Sky-ride" at the Chicago World's Fair (1933), and the Golden Gate Bridge have all been determined by model studies. These models have proved to be such a great aid in facilitating design and solving erection procedure problems that the company considers them indispensable on any major project.

Models have other uses, however, than those cited by Mr. Templin and in this discussion. Many structures have been designed either as a whole or in part as a result of stress analyses made with models. Probably the most extensively used system of model analysis is the deformeter method[136], which is particularly popular with State Highway, County and City Engineering Departments. The engineers in these departments are frequently charged with the design of relatively large and complicated, continuous, reinforced concrete structures.

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[136] "Indeterminate Structures Mechanically Analyzed", by George Erle Beggs, M. Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., Vol. 18, 1922.

The deformeter was used to advantage by Arthur G. Hayden, M. Am. Soc. C. E., in developing methods of design for the rigid frame bridge which was perfected by Mr. Hayden for use in grade separations in an extensive parkway system[137]. Six single-span, concrete, rigid-frame bridges of about 60-ft span, and two double-span, concrete, rigid-frame bridges were analyzed by this method. Each span of the double-span bridges was about 42 ft. After the construction program was well under way for this system of bridges, quick analytical methods were developed, which replaced the model studies. The model studies pointed the way to the efficient analytical method, and the accuracy of the method was established by comparison with the mechanical analyses.

The Missouri State Highway Department has made mechanical analyses as a part of its regular design procedure. The deformeter was applied in the Department's design of a self-anchored suspension bridge tower of the Vierendeel type. The tower was about 60 ft high and had all diagonal bracing omitted. This stress analysis was considered by the designing engineer of the State Highway Department to be more reliable than the tedious mathematical analysis. The results were very satisfactory, the cost being 0.1% of the cost of the structure. The latter was reduced by reason of the mechanical analysis to an extent which made the model study more than self-supporting.

The Missouri State Highway Department also used the deformeter in the analysis of a structure which consists of five reinforced concrete open-spandrel arches of 195-ft span with the deck structure continuous over each span. The two end spans were unsymmetrical. The purpose of the mechanical analysis was to show the effect of the deck structure. This deck structure was not considered in the mathematical analysis because the arches were on high piers and it would have been necessary to solve 285 simultaneous equations. The model was constructed of celluloid to represent the full length of the bridge. The scale chosen made the model about 14 ft long. Again, the results were satisfactory, the cost in this case being 0.2% of the cost of the structure, and the model was more than self-supporting.

Citation of other and varied examples of the usefulness of this device could be continued indefinitely.

Because it could be easily fabricated for the purpose, bronze was selected as the material for a model of the Bayonne Arch (Kill van Kull), the longest two-hinged spandrel-braced steel arch bridge in the world[138]. The model was built in 1929 under the direction of the consulting engineer and was used partly to check the results derived from theoretical investigations and partly to determine more specifically the importance of sway-bracing for distributing unsymmetrical loads between the two arch ribs, and the behavior of the portals in transmitting wind stresses. This model falls in a class mentioned by Mr. Templin as being different in some details from the prototype, but nevertheless very useful, and emphasizes the point which he made that models do not need to be exact scale reproductions, nor do they need to satisfy every theo-

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[137] "The Rigid Frame Bridge," by Arthur G. Hayden, John Wiley and Sons, 1931.

[138] "Design, Materials and Erection of the Kill van Kull (Bayonne Arch)," by Leon S. Moisseiff, M. Am. Soc. C. E., *Journal*, Franklin Inst., May, 1932.

retical requirement for exact similitude. Built-up sections of the full-sized structure were represented by solid members in the model. It was not possible, therefore, to reduce the area in the required proportion and, at the same time, assure equivalent stiffness against buckling. Areas of all members were reduced in the same ratio. The proportional behavior of the two structures was not affected by the departures from strict similitude. Static loads were applied to the model and the resulting strains and deflections were read with commercial instruments. The results obtained from this study were entirely satisfactory.

Welding, a method of fabrication which is increasing in popularity, has had a distinct effect in revealing and improving poor designs in some fields of construction. The newer designs have often presented difficult stress problems due to the shapes made possible by flame-cutting and the greater thicknesses which can be joined by welding. The safety of these new structures has been determined mainly by one of the following experimental methods: Testing of full-sized structures to destruction; non-destructive tests of full-sized structures in which the overstressed points are detected by the cracking of rosin or the flaking of a lime wash applied to the surface before testing; non-destructive tests in which strain-gage measurements are taken over the surface; and testing miniature models of like or of different material.

Many engineers feel that the use of miniature models in this field is not particularly fruitful. The departure of stress variation from the usually assumed straight-line distribution, the variable ability of ductility to adjust overstress in plates of different thicknesses, the relatively small size of the completed units, the residual stresses caused by the welding, and the new shop technique necessary to make a scalar reproduction, all tend to convince the producer of welded structures that he obtains more reliable information from a full-sized test.

In 1930, H. V. Spurr, M. Am. Soc. C. E., published[139] a method of design for wind bents in which he utilized a direct and rational method of design in contradistinction to the approximate-investigate-re-distribute-re-analyze methods common prior to that time.

As a check upon the Spurr method, a steel model was constructed and tested at the Experiment Station of Ohio State University[140]. The results of this investigation have been published as an authoritative study on the reliability of the Spurr method of design. The conclusions and recommendations are too numerous to mention herein, but the investigation serves as an excellent illustration of the usefulness of model studies in establishing the validity, or in making desirable revisions of practical, and somewhat approximate, design methods, so that they can be used with confidence by the profession. Without doubt this model, and the information obtained therefrom, will have a marked effect on the design of many of the buildings of the future.

The knowledge of wind pressures and velocities at surfaces of exposed structures is necessary to the successful application of any theory for the cal-

[139] "Wind Bracing," by Henry V. Spurr, McGraw-Hill Book Co., 1930.

[140] "Tests and Design of Steel Wind Bents for Tall Buildings," by G. E. Large, Assoc. M. Am. Soc. C. E., Samuel T. Carpenter, Jun. Am. Soc. C. E., and Clyde T. Morris, M. Am. Soc. C. E., Eng. Experiment Station, *Bulletin No. 93*, Ohio State University.

culation of wind stresses in members within the structure. In general, more thought must be given to necessary violation of the laws of similitude when dealing with dynamic or moving loads than when considering a static system. For this reason tests on the airship hangar at Lakehurst, N. J., conducted under the joint sponsorship of the Bureau of Yards and Docks of the U. S. Navy Department, The National Advisory Committee for Aeronautics, and Rensselaer Polytechnic Institute, should be of particular value.

Wind-tunnel tests were made on 1 to 800, 1 to 400, 1 to 200, and 1 to 40-scale models and actual pressure measurements are now (1936) being made on the full-sized hangar itself. The measurements on the full-sized hangar plus those made on the models will serve to provide a more reliable basis than has been available in the past for the evaluation of wind-tunnel tests on small models. The science of fluid dynamics, applied as it is at present, to problems involving all kinds of motion in any medium, owes much of its progress to the results of model studies.

Mention will be made of one type of test which is in the field of the Mechanical Engineer, but which also has a real interest for the Civil Engineer. Most of the monumental dams recently built and now under construction would have been impractical had it not been that equipment was being developed to utilize the hydraulic power which they made available. Elaborate equipment has been perfected for testing new designs of turbine rotors[141]. Models are made to include new features and are tested to determine their ability to resist fatigue stresses. The effective operation of the rotor requires irregularities of shape which make analytical calculations extremely difficult. On the other hand, the unit as a whole must meet continuous operating requirements. Failure of a rotor, revolving at present high speeds, cannot be contemplated—the design must be good. Inexpensive models may be allowed to fail without danger or inconvenience and the designer, in this way, is at liberty to try new and perhaps novel ideas. The publications of men interested in this field have repeatedly pointed out the usefulness of models of this type used in an extremely practical manner to determine fatigue limits of new materials or new designs.

Still another method of model investigation which should not go unmentioned is the method of testing underground structures which has been developed by Philip B. Bucky, Professor of Mining at Columbia University, in New York City. In this method the model is made an exact duplicate of the structure and strained by placing it in a centrifuge which, with the revolutions per minute properly regulated, produces centrifugal forces equivalent to the gravitational forces encountered in a sub-surface structure. The cost of an experimental analysis is small. Safe spans for flat and arched roofs, and the stress distribution within pillars, have been determined successfully in this manner by the combined use of stroboscopic and photo-elastic equipment. The practical application of this method requires a rather complete geological knowledge of the strata in which the proposed structure is to

[141] "Fatigue Tests of Model Turbo-Generator Rotors", by R. E. Peterson, *Mechanical Engineering*, March, 1931.

be built, together with a comprehensive and thorough understanding of theory and experimental technique.

Considerable experimental research work has been done by the faculty and graduate students of the universities. The Beggs deformeter, photo-elasticity, and loaded models have all been used extensively and from reports which have been gathered, it would appear that much effort has been wasted because many men have studied the same problem, only to have the results filed away and lost to the profession. These efforts, however, are not entirely wasted. A student will gain an understanding of structural action from working with models which might take years of study to acquire. Models, without doubt, serve a useful purpose when used to supplement class-room instruction. It is regrettable, nevertheless, that more co-ordinated and continued studies cannot be made. That this is possible is demonstrated by the tests which have been conducted on a miniature building frame by various students at Iowa State College under the direction of R. A. Caughey, M. Am. Soc. C. E. This miniature model has been revised and used repeatedly to solve various problems.

Illustrations of successful model studies could continue at great length, but the effort seems unnecessary. Mr. Templin has indicated the possibilities of making experimental studies; this discussion has attempted to show by means of actual illustration that experimental studies are more than possible. They are a much used tool of many present-day designers.

Models, however, are not just built. There are strict requirements of similitude. A real danger exists in that some designers may be led, by reports of startling results obtained with models, to believe that the model is a substitute for analytical ability. Such is definitely not the case. The successful model investigations have been made by engineers skilled in theoretical analysis, capable of designing a model, recognizing its limitations, and interpreting the experimental data. An engineer who has mastered the technique of using models has the ability to construct calculating machines which will assist him to design structures of unprecedented magnitude and importance. Mr. Templin has made a valuable contribution to engineering literature.

In an exceedingly vivid description of the photo-elastic method of determining stress, Mr. Brahtz raises an old point, which attains new importance in the design of the larger present-day structures, when he states that it is not always a simple matter to keep the probable error as low as 10 per cent. The writer is of the opinion that the results are very acceptable in the majority of cases if no greater discrepancies exist.

Engineers have always used a factor of safety. With loading conditions as unknown and variable as they are, this seems essential. However, the factor of safety has cared for three uncertainties: (1) Unknown loading conditions; (2) variability of the material which carries the loads; and (3) approximations in the design method. Human nature, and Nature herself, being what they are, there seems to be little hope of decreasing the factor of safety by reducing therein the requirements of Item (1). Much has been done to eliminate uncertainties in structural materials, and a conse-

quent increase in working stresses has resulted, thus reducing the requirements for Item (2). The "factor of ignorance" (Item (3)), seems to be the most vulnerable point of attack.

Many structures, costing millions of dollars each, have been built recently, and the designers, almost without exception, have been using to advantage the newly perfected methods offered by experimental analyses. In some cases, errors of appreciable magnitude have been discovered in accepted theoretical methods—in particular, those dealing with hyperstatic structures. In almost all cases the experimental results have furnished a confidence which fostered a more efficient design.

The writer doubts whether results are in general within 10% of the actual, even if they may be within a small percentage of that required by the assumed loads. Mr. Brahtz is to be commended for calling attention to the difficulties of technique involved in a reliable mechanical analysis. The photo-elastic method, as a practical tool, is both new and fascinating. As such, it is subject to both incorrect and too frequent use. The results obtained from the polariscope and other auxiliary devices certainly must be judged in the light of all factors, and not merely by the inherent accuracy of the method.

WERNER LEHMAN,[142] ESQ. (by letter).[142a]—Many phases of applications of aluminum alloys in the field of excavating machinery have been investigated by the writer, in which light weight structures are desirable in long booms, buckets, and other parts that must be counterbalanced and revolved at long radii. A reduction in weight of such parts is easily found to be economical through an increase in output or an increase in reach or working radius of the machines. The selling of such a benefit is often more difficult than the proof of economy.

Engineers are mostly interested in the capacity of a finished product to perform the intended purpose and for that reason the writer agrees with the importance of the questions presented by Messrs. Jeffries, Nagel, and Wood (see heading, "Effect of Cold Working"): (1) Will the new structure possess adequate strength? and (2) will it retain that strength under the anticipated service conditions?

The first question can be well defined by the engineer using the proper proportions for the characteristic of the building material. Structural Aluminum 17ST can be trusted as close to the values given in the data books as steel structures, provided the material is handled correctly in the process of manufacture so that no parts are changed in their characteristic properties by excess heat or local over-stress. With the proper instructions one can rely on the manufacturing department so that no deductions are made for improper handling. The aluminum alloys are not more difficult to handle than high-strength heat-treated steels. To avoid errors, the writer does not advocate any heating of aluminum in manufacturing processes.

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[142] Chf. Engr., Bucyrus-Erie Co., South Milwaukee, Wis.

[142a] Received by the Secretary, December 9, 1936.

Closely spaced rivets are driven at random to prevent local over-heating. Riveting with hot steel rivets has shown little ill effect on the heat-treated alloy, 17ST, used in long dragline booms. Most holes for bolts and rivets are drilled.

The second question is not so easily answered, because experience is not available over long periods as with steel. On an average, aluminum alloys can be trusted as well as steel. Concerning the structures of excavating machinery with their constant shock, vibrations, rapidly changing loads, and often occurring unforeseen strains, there is not enough knowledge of the long-time fatigue factor. It seems that occasional rest periods have the same healthy effect of aging and normalizing as in steel structures.

The writer agrees with Mr. Hartmann concerning the need for careful study of slenderness in compression members as a composite strut as well as the detail of the sections. Considerable test data are available. His statements concerning savings on long dragline booms agree with the writer's experience. The resistance factor of aluminum to corrosion does not enter into the designs and uses for excavation machinery. Abrasion resistance is more important in the design of buckets for excavators where it may be a deciding factor in the selection of material. The coefficient of expansion and the modulus of elasticity are both unsuitable to combine aluminum with steel in the same section.

Probably the strongest competitor to light weight alloys is the progress in the art of welding of steel and the recently developed weldable steel alloys of great strength and resistance to abrasion. The peculiar shapes of dippers and drag-buckets does not lend itself to extensive economical use of aluminum mainly because the resistance to abrasion and shock is all important and the use of wearing plates defeats the benefit derived from light weight alloys.

When saving of weight is important, any designer of machinery should keep the excellent properties of structural heat-treated aluminum alloys in mind and should weigh their possibilities not only in the detail, but as to their effect on the complete structure.

OTIS E. HOVEY,[143] M. AM. SOC. C. E. (by letter).[143a]—In the Specifications for Steel Railway Bridges of the American Railway Engineering Association, dated August, 1935 (in Part III, "Alloy Steels, Foreword", p. 49, Article 9), it is stated that "no steel should be considered satisfactory for bridge construction if its yield point exceeds 70 per cent of the ultimate strength." In some cases, this rule has been quoted as an authoritative requirement for all structural alloy steels and particularly with reference to the low-alloy steel the chemical and physical properties of which are given by Messrs. Bain and Llewellyn (see heading, "Complex Steels: Simultaneous Addition of Several Elements"). It will be interesting to trace the origin and application of the limiting yield point ratio of 70 per cent.

During an extensive research into the properties of heat-treated carbon-steel eye-bars, in 1914, the late C. G. Emil Larsson, M. Am. Soc. C. E., and

[143] Cons. Engr., New York, N. Y.

[143a] Received by the Secretary December 21, 1936.

the writer studied the results of many tests with respect to physical properties developed and with particular reference to ductility. The suggestion to limit the yield point ratios to 70% of the ultimate tensile strength was made for the purpose of insuring the desired ductility in the finished product. It should be remembered that carbon steel, and not alloy steel, was under investigation at that time. At a later date the same limiting yield point ratio was inserted in the Specifications of the American Railway Engineering Association for alloy steels.

The paper by Messrs. Bain and Llewellyn shows that there are several combinations of low-carbon steel with various alloying elements that produce low-alloy steels which have satisfactory ductile properties, even if their yield points in some cases are greater than 70% of their ultimate tensile strengths.

The writer believes that arbitrary rules limiting the yield point ratio should not be made, but that attention should be given to the ductile properties, the endurance limits, and to the impact properties, in order to be sure that a particular steel will be suitable for bridges and other structures.

Mr. Moisseiff has presented a paper of much interest and great value at a time when many engineers are keenly interested in the development of high-strength steels appropriate for use in heavy and long-span bridges and in other structures in which a stronger steel is needed.

The presentation of this paper brought to mind a matter of some historic interest, since it deals with the evolution of what may be termed low-alloy high-strength steels. As a matter of fact the steel first used in the principal truss members of a long-span bridge was an alloy. This refers to the trussed arches of the first Mississippi River Bridge, at St. Louis, Mo., commonly known as the Eads Bridge. The main members of the arches are hollow cylinders about 18 in. in diameter, in panel lengths, and each section is made up of six steel staves bound together by outside cylindrically formed plates. The material in the staves is a chrome alloy steel.

The contract for the superstructure, erected in place, was signed on February 26, 1870. The celebration of the completion of the bridge occurred on July 4, 1874. During 1870 and 1871 an investigation was made of various steels, which resulted in the choice of the chrome alloy steel. A study of the specifications reveals that the modulus of elasticity was considered to be one of the most important physical properties of the steel. A few quotations are given herewith:

"The modulus of elasticity of the steel in the staves shall not be less than 26 000 000 nor more than 30 000 000 pounds. This variation should be avoided if possible, in which case the lower amount will be preferable. Each bar will be tested by the contractor, and the modulus stamped on it by the inspector. * * *."

"Each tube is to be composed of six staves having, as near as possible, the same modulus of elasticity."

"One specimen bolt from each twenty staves will be made by which to test their limits of elasticity and ultimate strengths in tension."

"The specimen bolts must be able to sustain a tensile strain of 40 000 pounds per square inch, without permanent set, and must have an ultimate strength of 90 000 pounds per square inch of section."

Elsewhere, it is stated that "should some of the sections of a tube when under compression yield more than others, it is obvious that those others would be forced to bear more than their share of the load." Furthermore, "small specimens of the steel were, however, continually tested to destruction to ascertain both the modulus of elasticity and the ultimate strength. The modulus was kept at about 27 000 000 and the ultimate strength at about 120 000 pounds per square inch." The stress-strain line of a full-sized test of a stave was nearly straight up to a compressive unit stress of 50 321 lb per sq in. "In compression almost any degree can be obtained by the addition of chrome. To avoid difficulty, however, in finishing the steel in the lathes, it is only made sufficiently hard to meet the requirements of the specifications." A supplementary contract provided that "the staves composing the tubes shall not be required to stand a compressive strain exceeding 50 000 pounds, or a tensile strain exceeding 40 000 pounds, per square inch of section, without permanent set."

At a time when there is so much interest in the development of alloy steels, particularly adapted to special applications, the writer deems it proper to call attention to the attitude of Captain Eads and his staff, 66 yr ago, toward the use of alloy steel in the first long-span bridge, which now carries live loads far in excess of those for which it was designed. Much additional interesting data may be found in "A History of the St. Louis Bridge", by Professor Calvin Milton Woodward, published in 1881.

R. G. Sturm,[144] Assoc. M. Am. Soc. C. E. (by letter).[144a]—In the various papers presented in this Symposium many problems confronting the users of new materials in structures have been presented. Special emphasis should be given to a consideration of the stability problem, several phases of which have been treated in the Symposium. The writer wishes to connote a conception of stability in the more general sense; that is, stability against buckling, stability against vibration, stability against shock, and stability against excessive deformation with time. In connection with this general definition of stability, it is proposed to consider the actual volume of the metal (without relation to weight) that goes to make up a given member of any structure, and the effect of that volume on each of the aforementioned types of stability.

Stability against buckling is influenced not only by the modulus of elasticity of the material, but also by the shape and proportions of the cross-section of the member. The member must be strong enough not only to carry its load as a whole, but each component part must also be able to carry its share of the load without local buckling. Since the stress at which local buckling occurs varies as the square of the thickness, it is possible that the thickness of the material may influence the stability more than the modulus of elasticity. Structural members often fail by twisting, in which case the torsional rigidity of the member plays an important part in its stability.

[144] Research Engr. Physicist, Aluminum Co. of America, New Kensington, Pa.

[144a] Received by the Secretary January 2, 1937.

The torsional rigidity of thin open members varies as the cube of the thickness of the material. It may be concluded from these considerations that in some cases the actual volume of the member might have a greater effect than the modulus of elasticity.

Stability against vibration is achieved by two means: First, the natural frequency of vibration may be made so high that resonance or near resonance will not occur in practice; and, second, if resonance is approached, the damping effect of the system may be made great enough to prevent excessive amplitudes of vibration. The frequency of vibration of any member is greatly increased by the rigidity of its end connections, and the rigidity of a connection is influenced greatly by the torsional rigidity of the members framing into the joint. Therefore, in this case also, the actual thickness or the volume of the material is an important factor in stability against vibration.

In general, the natural frequency of vibration of a given structure may be fairly well approximated by the equation:

$$\text{Frequency} = \frac{3.13}{\sqrt{\text{Static deflection}}} \quad \ldots\ldots\ldots\ldots\ldots (26)$$

One finds that the weight and modulus included with the design control the static deflection of the structure under its own weight and, hence, its natural frequency. Resonance between applied impulses and the natural frequency of a structure is one of the greatest factors causing dynamic effects of high-speed traffic on large bridges.

Stability against shock is a difficult quantity to measure. Elastic resistance to shock is greatly increased by lowering the natural frequency of vibration, which is just the opposite of the condition sought for to provide stability against vibration. Therefore, a balance between these two conditions must be sought. In general, it is desirable to have members that will withstand emergency shocks without complete fracture even if they do take a permanent set. Very thin sections generally tend to tear or buckle, whereas heavier sections can deform plastically and take considerable permanent set without a complete fracture Therefore, the stability against shock will be influenced greatly by the volume of the material in the member as well as by other factors. Another factor very significant in resisting shock is the ductility of the material considered. Ductility is intended herein to connote the capacity of the material to become elongated over relatively long lengths rather than to "neck down" at a local point.

The stability against excessive deformation with time requires a balance between the beneficial effects and the deleterious effects of creep. High concentrated stresses are unavoidable at some localities in structures. If, by virtue of a slight amount of creep, the metal can adjust itself, these concentrated stresses are distributed, and the danger of failure at that point is greatly reduced. If the material at the point of concentrated stresses is not thick enough to permit such a redistribution of stress, minute buckling or tearing through local instability may occur before creep has an opportunity to take place. In such event, the actual volume of the material at the point of

highly stressed concentration is of a great significance. In general, over-all deformation of structural members under stress can be controlled by a proper choice of design stresses for the material considered, whereas the local deformations around joints cannot be so controlled.

Many of the foregoing factors have been considered in the Symposium but is seems well to emphasize the fact that the general stability of a structure may be influenced greatly by the thickness of the material in the members of the structure and that with materials of low specific gravity it is possible to obtain a lighter dead weight and still maintain adequate thickness to provide stability.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

STRESSES AROUND CIRCULAR HOLES IN DAMS AND BUTTRESSES

Discussion

BY MESSRS. R. D. MINDLIN, AND CHESLEY J. POSEY

R. D. MINDLIN,[7] JUN. AM. SOC. C. E. (by letter).[7a]—In examining Equations (28), (29), (30), and (34) for the stresses around a circular hole resulting from the gravitational action of a dam, it is observed that the elastic constants of the material are not involved. This is surprising in the light of a recent extension by M. A. Biot[8] of a theorem by J. H. Michell[9].

Michell's theorem applies to a two-dimensional stress system in a region pierced by holes, no body forces being present. He showed that in such a system the stresses are independent of the elastic constants only if the resultant force over each hole vanishes. This conclusion is found necessary in order to assure single-valuedness of the displacements.

Biot has extended Michell's theorem to the case of gravity stresses. He has shown[8] that "in a solid two-dimensional homogeneous body the stresses due to gravity do not depend on the elasticity constants of the material; they do, however, in general for a body with holes." Therefore, a more detailed examination of the stresses due to mass is indicated.

The author has conveniently divided the stresses in the solid dam into two systems. In System I the stresses on any closed boundary produce zero resultant force. Hence, the additional stresses required to remove the normal and shearing stresses on the periphery of a circular hole also involve zero resultant force on the hole. No difficulty with many valued displacements is encountered, therefore, in dealing with System I, and the author has found the required stress system by a well known method.

NOTE.—The paper by I. K. Silverman, Jun. Am. Soc. C. E., was published in November, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[7] Research Asst., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

[7a] Received by the Secretary November 21, 1936.

[8] "Distributed Gravity and Temperature Loading in Two-Dimensional Elasticity Replaced by Boundary Pressures and Dislocations", by M. A. Biot, *Journal of Applied Mechanics*, Vol. 2, No. 2 (June, 1935), p. 41.

[9] "On the Direct Determination of Stress in an Elastic Solid, with Application to the Theory of Plates", by J. H. Michell, *Proceedings*, London Math. Soc. Vol. XXXI (1899), p. 100.

In dealing with System II, Mr. Silverman has observed that these stresses produce a resultant force on an internal circular boundary equal to the weight of the enclosed disk. In order to remove the stresses from the periphery of the hole, an additional stress system must be superposed which yields equal and opposite stresses on the circular boundary. The latter system, then, must involve a resultant stress on the hole equal and opposite to the weight of the enclosed disk of radius, r_h. Therefore, according to Michell's theorem, the superimposed system should involve the elastic constants. This was to be expected from Biot's theorem. In fact, the foregoing explanation is simply a derivation of Biot's theorem for the present case.

If the elastic constants do not appear in a case where the resultant force on an internal boundary does not vanish, then, by Michell's theorem, the displacements resulting from the stresses should be many valued. Furthermore, when the internal boundary is closed, as in this case, many-valued functions in the displacements are not permissible, and a solution containing them is not valid.

It is necessary, therefore, to proceed to examine the stress function (Equation (33)) which was used to annul the stresses on the boundary, $\rho = r_i$. Writing $c\, r^2_i = a_1$, Equation (33) becomes[9b]:

$$F_1 = \frac{a_1}{2} \rho\, \theta \sin \theta \quad \ldots\ldots (50)$$

The stress function given in Equation (50) is known to produce many-valued functions in the displacements of the form[10], $\frac{a_1}{2} \frac{(1 - \nu)}{E} \theta \sin \theta$; and $\frac{a_1}{2} \frac{(1 - \nu)}{E} \theta \cos \theta$, respectively, in which ν = Poisson's ratio. This was to be expected from the preceding analysis, and it may be concluded that the stress system represented by Equations (34) is not possible in a continuous elastic medium.

To obtain the correct system of stress required to remove the stresses on the periphery of the hole, a bi-harmonic stress function must be found which will satisfy the following conditions:

(1) The displacements introduced by the stress function must be single-valued;

[9b] Correction for *Transactions*: The author has written θ for $(\theta - \beta)$. This should be kept in mind when adding the stresses involved in System II to those of System I. The following corrections, also approved by the author, will be made before the paper is finally published in *Transactions*: In the line below Equation (8), change "Equation (1)" to "Equation (6)"; Equation (10) should read "$s_\rho = - \frac{\rho}{6 K^3} [\ldots\ldots$"; in Equation (25), "$K_2$" should be "$k_2$"; to Equation (27*e*), add "$= B_3 = C_3$"; in the line following Equation (32), change "θ" to θ'; change Line 2 following Equation (32), to read "$(s_\theta)_{II} = c \left(\frac{r^2_i}{r^2_o} - 1\right) \rho \cos \theta'$; $(s_s)_{II} = 0$; and, $\phi' = \phi - \beta$"; in Line 3 following Equation (34*c*), write "$s_\theta = - c r_h \cos \theta$"; in Equation (36), change "+0.00204" to read "+0.02204"; in Equation (37), insert a minus sign before 1.13; in Table 1, for $\theta = 180$, $s_\theta = -1.5877$; in Appendix I, delete the definition for K_2 and add the definition "$k_2 = \cot \alpha$" in the definition for k_n; and, in Equation (45*a*), insert a minus sign before "$G_n\, q_n$".

[10] See "Theory of Elasticity", by S. Timoshenko, p. 116, McGraw-Hill, 1934.

(2) The normal and shearing stresses on the circular boundary, $\rho = r_i$, must be equal and opposite to those of System II; and,

(3) All the stresses introduced by the new function must approach zero as ρ approaches infinity.

In order to satisfy Condition (1) one must combine with Equation (50) a stress function[10],

$$F_2 = b'_1 \rho \log \rho \cos \theta \qquad (51)$$

in which,

$$b'_1 = -\frac{a_1 (1 - \nu)}{4} \qquad (52)$$

The combined functions, F_1 and F_2, satisfy Conditions (1) and (3), but not Condition (2). The addition of F_2 to the system has served to introduce shearing stresses on the circular boundary (proportional to $\sin \theta$), which are not present in System II or in F_1. Therefore, another function, F_3, must be added, which will remove these shearing stresses without introducing normal stresses on the circular boundary other than those proportional to $\cos \theta$. Furthermore, F_3 must satisfy Conditions (1) and (3). Such a function is given by:

$$F_3 = a'_1 \rho^{-1} \cos \theta \qquad (53)$$

The complete stress function is then given by:

$$F = F_1 + F_2 + F_3 = \frac{a_1}{2} \rho \theta \sin \theta + b'_1 \rho \log \rho \cos \theta + a'_1 \rho^{-1} \cos \theta \qquad (54)$$

To determine the values of the constants it is necessary first to calculate the stresses. Using Equations (20) of the paper, these are found to be:

$$s_\rho = \left(\frac{a_1}{\rho} + \frac{b'_1}{\rho} - \frac{2 a'_1}{\rho^3} - c \rho\right) \cos \theta \qquad (55a)$$

$$s_\theta = \left(\frac{b'_1}{\rho} + \frac{2 a'_1}{\rho^3} - c \rho\right) \cos \theta \qquad (55b)$$

and,

$$s_s = \left(\frac{b'_1}{\rho} - \frac{2 a'_1}{\rho^3}\right) \sin \theta \qquad (55c)$$

Condition (2) is expressed by the equation:

$$[s_\rho]_{\rho = r_i} = [s_s]_{\rho = r_i} = 0 \qquad (56)$$

and, hence:

$$\frac{a_1}{r_i} + \frac{b'_1}{r_i} - \frac{2 a'_1}{r^3_i} - c r_i = 0 \qquad (57a)$$

and

$$\frac{b'_1}{r_i} - \frac{2 a'_1}{r^3_i} = 0 \qquad (57b)$$

Condition (1) is expressed by Equation (52). Solving for the three unknowns in Equations (52) and (57):

$$a_1 = c\, r^2_i \qquad (58a)$$

$$b'_1 = -\frac{1-\nu}{4}\, c\, r^2_i \qquad (58b)$$

and,

$$a'_1 = -\frac{1-\nu}{8}\, c\, r^4_i \qquad (58c)$$

Substituting these values in Equation (54) the final stress function becomes:

$$F = \frac{1}{2}\, c\, r^2_i\, \rho\, \theta \sin\theta - \frac{1-\nu}{4}\, c\, r^2_i\, \rho \log\rho \cos\theta - \frac{1-\nu}{8}\, c\, r^4_i\, \rho^{-1} \cos\theta \qquad (59)$$

The corresponding stresses are, from Equations (20):

$$s_\rho = c\left(\frac{3+\nu}{4}\,\frac{r^2_i}{\rho} + \frac{1-\nu}{4}\,\frac{r^4_i}{\rho^3} - \rho\right)\cos\theta \qquad (60a)$$

$$s_\theta = -c\left(\frac{1-\nu}{4}\,\frac{r^2_i}{\rho} + \frac{1-\nu}{4}\,\frac{r^4_i}{\rho^3} + \rho\right)\cos\theta \qquad (60b)$$

and,

$$s_s = c\left(-\frac{1-\nu}{4}\,\frac{r^2_i}{\rho} + \frac{1-\nu}{8}\,\frac{r^4_i}{\rho^3}\right)\sin\theta \qquad (60c)$$

Equations (60) express the correct stresses for System II. By virtue of Equation (52), Condition (1) is satisfied; by virtue of Equations (57*a*) and (57*b*). Condition (2) is satisfied; and an examination of Equations (60) shows that Condition (3) is satisfied. The hoop stresses at the circular boundary are found by substituting $\rho = r_i$ in Equation (60*b*)[9b]:

$$[s_\theta]_{\rho=r_i} = -\left(\frac{3-\nu}{2}\right) c\, r_i \cos\theta \qquad (61)$$

The foregoing analysis has been made for the case of plane stress. It is approximately applicable to a buttress if the diameter of the hole is small in comparison with its distance from the nearest edge of the buttress, and if the thickness of the buttress is small in comparison with the diameter of the hole. For the case of a longitudinal hole in a gravity section the stresses should be transformed to the corresponding system for plane strain.

Additional references should be given to the results of previous stress analyses for the wedge section. The stress systems represented by Equations (8) to (12) and (22) to (25), inclusive, are due to M. Levy[11] and may be obtained from equations given by Love[12]. This stress system applies only to

[11] "Sur l'équilibre elastic d'un barrage en maçonnerie à section triangulaire," by M. Lévy, *Comptes Rendus*, Paris, Vol. 127 (1898), p. 10.

[12] "Theory of Elasticity", by A. E. H. Love, Fourth Edition, 1927, Equations (344) and (34*B*), pp. 212 and 213.

portions of a dam remote from the base. Hence, in addition to the requirement that the small hole be far from the faces of the dam, it must also be far from the base.

Equations (8) and (22) are seen to be of the same form, differing only in the constants associated with the terms of the third degree polynomial. Equation (22) represents the stress which the author has designated as System I. Hence, the additional stress functions, required to annul the stresses due to System I on a circular boundary, will be the same as the stress functions required to annul the stresses on the circular boundary resulting from the water loading, except for the constants. Therefore, a solution of the problem of the stresses due to mass really includes that of the problem of stresses due to water load.

The author is undoubtedly unaware of a previous solution of this problem by G. Nishimura and T. Takayama[13]. Of the two solutions, the writer prefers that of Mr. Silverman as being far less cumbersome. Very skilfully the author has applied what mathematicians would call an elegant method and, independently, has obtained the solution of an interesting problem.

CHESLEY J. POSEY,[14] JUN. AM. SOC. C. E. (by letter).[14a]—The author is to be commended for presenting an analytical proof of the existence of tensile stresses near longitudinal circular holes in concrete gravity dams. The question of the proper reinforcement around holes is of importance in many types of concrete design. Although most designers would provide extra reinforcement around a hole in a girder web, many would not admit that any was necessary for a long, relatively small hole in mass concrete. Mr. Silverman's analysis, made on the basis of reasonable assumptions, shows that such reinforcement is necessary for longitudinal holes in gravity dams, to resist stresses due to water and mass loads. In some cases, stresses around the openings, due to volumetric changes from shrinkage and temperature, may call for even more reinforcement.

Using a method of attack similar to that presented by Mr. Silverman, the writer believes it would be possible to show that, in general, circumferential tensile stresses will occur around at least part of the periphery of a long straight hole in mass concrete, if the state of stress in the concrete is such that shear exists on any plane containing the axis of the hole.

The proper placing of steel bars around holes is of great importance. The steel should never follow around the inside of a hole, for if it is stressed in tension it will tend to straighten, causing tension in the concrete. As soon as this concrete fails, the steel is useless, and cracking is perhaps further advanced than if no steel had been used. Fig. 6(a) is an example of incorrect, and Figs. 6(b), 6(c), and 6(d) of correct, placing of the reinforcement around holes. The placing shown in Fig. 6(d) is preferable to that shown in Fig. 6(c),

[13] "On the Stress Distribution in the Vicinity of a Horizontal Circular Hole in a Gravitating Wedge-Shaped Elastic Solid", by G. Nishimura and T. Takayama, *Bulletin*, Earthquake Research Inst., Tokyo, Vol. 10 (1932), p. 723.

[14] Asst. Prof., Mechanics and Hydraulics, State Univ. of Iowa, and Assoc. Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

[14a] Received by the Secretary December 2, 1936.

however. Diagonal bars should be provided at the corners of rectangular holes, for cracks tend to form at the corners, where vertical and horizontal bars are relatively less effective than diagonal bars. If it is certain that tensile stresses can never occur across one of the diagonal sections, the bars crossing that section may be omitted.

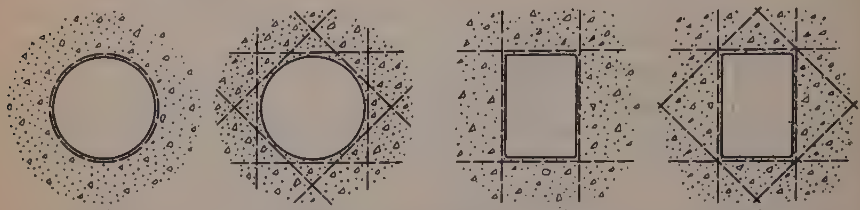

FIG. 6.—METHODS OF PLACING STEEL REINFORCEMENT AROUND HOLES IN MASS CONCRETE

Referring to Fig. 5 of the paper, it is seen that the tensile stresses decrease very rapidly away from the surface of the hole. The steel bars, therefore, should be as close to the surface of the hole as is practicable. The length of bar to be used cannot be computed by any method known at present, and, in the absence of test data, it must be arbitrarily assumed, say, equal to twice the diameter of the hole. The size of bar should then be selected so that the total bond on one-half its length is equal to or greater than its ultimate strength. To prevent cracks, the bar should have as high a ratio of bond stress to slip as is possible. Tests[15] by the writer show that the most favorable ratio is obtained if the bars are straight, and if their surface is rough.

[15] "Tests of Anchorages for Reinforcing Bars", *Bulletin 3,* Univ. of Iowa Eng. Studies.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

EQUITABLE ZONING AND ASSESSMENTS FOR CITY PLANNING PROJECTS

PROGRESS REPORT OF COMMITTEE OF CITY PLANNING DIVISION

Discussion

By R. S. Blinn, M. Am. Soc. C. E.

R. S. Blinn,[7] M. Am. Soc. C. E. (by letter).[7a]—Adhering to the six principles operative in the field of special assessments and casting aside the methods of assessments by the foot front and by a percentage of the tax value as entirely empirical and unequitable, there remains the equitability of assessments by benefit.

On closer analysis this seems to be as unequitable as the other methods: First, because a scientific appraisal of each property would be necessary, then a correlation of all the increments that make up an appraisal distributed over all the property to be assessed; and, further, an accurate knowledge of the economic trend during the life of the assessments. Who knows what that will be? And, second, because the benefits may be a minus quantity and may result in an expense instead of an investment. It is axiomatic that net income is capitalized into net worth whereas net deficit is capitalized into net loss.

This leads to the logical conclusion that all assessments by whatever method are vulnerable, so that unless the property holders wish to waive their rights under the law and do the work voluntarily, there is no other recourse to raise the necessary funds to make an improvement, except through general taxation under the six principles first mentioned. In this way the gamble as to its worthwhileness becomes a gamble of the entire community.

Note.—The Progress Report of the Committee of the City Planning Division on Equitable Zoning and Assessments for City Planning Projects was presented at the meeting of the City Planning Division, New York, N. Y., January 15, 1936, and published in February, 1936, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: August, 1936, by George H. Herrold, M. Am. Soc. C. E.

[7] New Smyrna, Fla.

[7a] Received by the Secretary, September 29, 1936.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

FLOOD PROTECTION DATA

PROGRESS REPORT OF THE COMMITTEE

Discussion

BY C. H. EIFFERT, M. AM. SOC. C. E.

C. H. EIFFERT,[21] M. AM. SOC. C. E. (by letter).[21a]—The work of the Committee is very important. The compilation and analysis of all existing data are essential to the proper solution of flood-control problems. The data and records of the past form the foundation for any plans that must be developed in the immediate future. Although they are of rather short duration and of a somewhat fragmentary nature, these data are all that engineers have and should be made available in such a way that they can be used to the best possible advantage.

In addition to the collection, tabulation, and analysis of all pertinent existing data, the writer would like to see this Committee take a more emphatic stand concerning the improvement of the methods for the future collection of data and keeping of records. There are two ways in which improvement could be accomplished:

(1) By the re-organization and combination of the various Federal bureaus or departments, or sub-divisions thereof, which are now collecting such data, into one bureau, or by more complete correlation and co-operation of the existing bureaus; and,

(2) By the extension, expansion, and amplification of the financial abilities and physical facilities of the existing organizations so as to make possible a comprehensive, far-reaching, and permanent program.

Probably the most desirable step to be taken first would be the centralization or combination of all Government agencies that have to do with meteorological and hydrological records. If this is impossible of accomplishment in

NOTE.—The Progress Report of the Committee on Flood Protection Data was presented at the Annual Meeting, New York, N. Y., January 15, 1936, and published in February, 1936, *Proceedings*. Discussion on the report has appeared in *Proceedings*, as follows: April, 1936, by Messrs. Robert Follansbee, and LeRoy K. Sherman; August, 1936, by C. R. Pettis, M. Am. Soc. C. E.; September, 1936, by Messrs. John C. Hoyt, and C. S. Jarvis; November, 1936, by Messrs. Gordon W. Williams, Merrill Bernard, and Glenn W. Holmes; and January, 1937, by Messrs. Charles D. Curran, and Edward N. Whitney.

[21] Chf. Engr. and Gen. Mgr., The Miami Conservancy Dist., Dayton, Ohio.

[21a] Received by the Secretary December 5, 1936.

the near future, the nearest approach to it will be the best possible co-operative arrangement that can be devised.

The necessity for the extension of physical facilities and the need for additional financial support should be evident to any one who has used meteorological records and who has come in contact with the representatives of the departments responsible for them. Most of these men are working hard and conscientiously to make $1 do where they should have $5 or $10. They are not lacking in ability either. Legislative support for the U. S. Weather Bureau and U. S. Geological Survey has been sadly deficient both from the Federal and the State Governments. For instance, in 1935, the U. S. Geological Survey program in Ohio, under which a splendid start had been made in the collection of run-off and stream-discharge data, was wrecked by the Governor's veto.

Records in the United States are in their infancy; they have been kept long enough so that now hydrologists know about what they really need. They owe it to future generations to put that knowledge to work in establishing an adequate system. Once in operation it must be kept going perpetually. The breaking off of records at one location, or at a certain time, and beginning them again at another location, is not satisfactory. It may cause designers to miss a 100-yr, or perhaps a 1 000-yr, flood or rainfall. If a flood-control system is designed on the basis of existing records (which, of course, it must be if it is designed at all), the engineer cannot assume that he needs no further information. He must continue to secure records to check the design and make changes if they are found necessary "Eternal vigilance" is the watchword.

Thousands of rain-gages are needed in the United States. Many should be of the recording type. The area of the United States is 3 027 000 sq miles, With approximately 4 500 Weather Bureau rain-gages, the average is 1 for every 670 sq miles. In the Po River Valley, in Italy, engineers have practised flood control and kept records for several hundred years, but they do not feel that their records are now sufficient so that they may be discontinued. They have added facilities in recent years. The area of the Po drainage is 27 500 sq miles and contains 917 rain-gages, 1 for every 30 sq miles. The area also has 137 river-gages, 66 discharge gaging stations, and 123 stations for measuring the ground-water level.

In other parts of Europe engineers have collected records for several centuries, and these records are becoming more and more valuable. On the Danube, flood records have been kept since 1 000 A. D. The greatest flood of record occurred in 1501. A record of 100 or 200 yr was not sufficient to determine the maximum flood. The flood of 1899 in the Danube was the greatest in more than a century, but the flood of 1501 had a 33% greater maximum discharge. Records comparable to that of the Danube are in existence for other European streams.

In the Miami Valley of Ohio hydrologists feel that they are quite well equipped. The U. S. Weather Bureau and the Miami Conservancy District together have 30 rain-gages (2 recording) on an area of 3 670 sq miles, or 1 for every 120 sq miles. This distribution is still far behind that of the Po

Valley. Gages are not needed all over the United States, spaced as closely as those in the Po Valley, but there are areas where they could be placed, one for every 30 sq miles, to good advantage, and they should be increased in all parts of the country.

Many engineers have complained about the inadequacy of Weather Bureau records, forecasts, and flood warnings. It is true that some of the methods in use by the Weather Bureau should and could be improved without additional funds, as, for instance, the present lack of uniformity in the time of observations. However, there can be no doubt that the principal difficulty lies in the lack of financial support. This was brought out quite forcibly by representatives of the U. S. Weather Bureau at the meeting of the Society in Pittsburgh, Pa., on October 15, 1936. The Weather Bureau authorities apparently are ready and more than willing to proceed with snow surveys, improved methods of taking observations and making forecasts, using the radio for communication, etc., provided the needed funds are available. The U. S. Geological Survey finds itself in a similar predicament.

It seems to the writer that there is no more important service that this Committee could render than to use its influence and that of the Society to the limit to bring about the necessary legislation for the proper support of an adequate permanent program for the collection of meteorological data in the future. The data would be useful and valuable for many studies other than those of flood control. The cost would be negligible in proportion to the ultimate benefit.

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DISCUSSIONS

PRINCIPLES TO CONTROL GOVERNMENTAL EXPENDITURES FOR PUBLIC WORKS

FIRST PROGRESS REPORT OF COMMITTEE OF ENGINEERING-ECONOMICS AND FINANCE DIVISION

Discussion

BY RAYMOND J. ROSENBERGER, ASSOC. M. AM. SOC. C. E.

RAYMOND J. ROSENBERGER,[30] ASSOC. M. AM. SOC. C. E. (by letter).[30a]—In general, the writer agrees most heartily with the principles stated in this report. However, he wishes to present a few suggestions for the consideration of the Sub-Committee. First, the name suggested for the proposed programming body is the "United States Program Authority." The writer recommends instead the "Federal Public Works Program Authority", or some similar wording more indicative of the function of the proposed agency. The jurisdiction of the Authority, could be Federal in scope, with similar Authorities under each State for non-Federal work. The Federal Authority could function along the details of procedure followed by the Sub-Committee of the Committee of the House of Representatives on Appropriations.

There seems to be no serious objection to a life term for members of the Authority. Expert aids and investigators could do the more active work, such as traveling, etc., these men to be selected by the members of the Authority from trusted and confidential acquaintances. A clarification of terms used in the seven broad principles by definitions and explanations would add considerably to the comprehensiveness of the report.

NOTE.—The Progress Report of the Committee of the Engineering-Economics and Finance Division on Principles to Control Governmental Expenditures for Public Works, was presented at the Annual Meeting, New York, N. Y., January 15, 1936, and published in February, 1936, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: April, 1936, by Messrs. Edward W. Bush, Fred Lavis, and Horace H. Sears; August, 1936, by Messrs. Ivan C. Crawford, and Samuel B. Folk; and September, 1936, by Messrs. J. K. Finch, Albert Givan, Edward Hyatt, Clarence H. Kromer, A. G. Mott, and C. S. Pope, and Joseph Jacobs.

[30] Engr.-Examiner, PWA, Washington, D. C.

[30a] Received by the Secretary September 19, 1936.

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DISCUSSIONS

DEFLECTIONS BY GEOMETRY

Discussion

By Messrs. A. J. McGaw, and L. E. Grinter

A. J. McGaw,[4] Jun. Am. Soc. C. E. (by letter).[4a]—The author has focused much needed attention upon the principles of geometry as they affect structural action. Although equally accurate results may be obtained by the methods embracing the law of conservation of energy and the principles of least work, and although these methods are in many instances desirable as analytical procedures, they do side-track the geometry involved, which is so essential to complete understanding of the subject of indeterminacy. As a teaching tool, at least, the geometric method has much in its favor. The physical conception of structural deformation and the part played by geometry must, of necessity, go hand in hand. The formula-minded analyst, in many cases, fails to see his structure act. It is probably safe to state that analyses made under such conditions are dangerously likely to contain error. There is little to fear from the designer who can sketch the structure at hand in its deformed shape. It follows that an understanding of the principles of geometry as they affect structural action is highly desirable.

Along this same line of thought, it seems that the development of fundamental theory, through the utilization of geometric principles, may be extended advantageously to include Maxwell's law of reciprocal deflections. Surely, any haziness in one's mind concerning the truth and logic of this fundamental and much used tool will prevent, to a considerable extent, real mastery of the subject of indeterminacy. With this in mind, the writer has adapted the reasoning presented by the author[4b] to evolve a proof of Maxwell's law.

As in the paper, the beam, AB, in Fig. 14(a) has been assumed rigid, except at Point G. Due to the unit load at Point D, the moment, M_G, at

Note.—The paper by David B. Hall, Assoc. M. Am. Soc. C. E., was published in December, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

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[4a] Received by the Secretary January 4, 1937.

[4b] The following corrections, approved by the author, will be made before the paper is published in *Transactions:* In Equation (9) include EI under the integration sign; in Equation (11), change "θ" to "ϕ"; in Line 1, p. 1536, include EI under the summation sign; and delete Fig. 12 and Exercise (2) adjusting figure numbers accordingly.

Point G, has produced a rotation at that point through the angle, α_D, equal to:

$$M_G = \frac{x\ a}{L} \qquad \text{(17)}$$

and since α_D is proportional to M_G,

$$M_G\ K = \alpha_D = \frac{x\ a\ K}{L} \qquad \text{(18)}$$

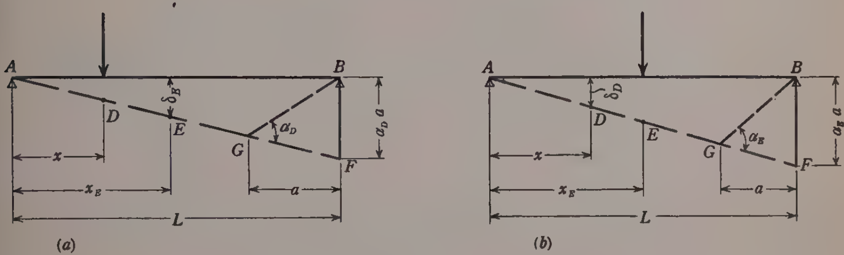

FIG. 14

Furthermore, the ordinate, BF, in Fig. 14, equals,

$$\alpha_D\ a = \frac{x\ a^2 K}{L} \qquad \text{(19)}$$

and the deflection at Point E equals:

$$\delta_E = \frac{\overline{BF}\ x_E}{L} = \frac{x\ a^2\ x_E\ K}{L^2} \qquad \text{(20)}$$

Similarly, the deflection at Point D (Fig. 14(b)), with a unit load at Point E, may be obtained as follows:

$$\alpha_E = M_G\ K = \frac{x_E\ a\ K}{L} \qquad \text{(21)}$$

and, the ordinate, BF, is equal to:

$$\alpha_E\ a = \frac{x_E\ a^2 K}{L} \qquad \text{(22)}$$

and,

$$\delta_D = \frac{\overline{BF}\ \ x}{L} = \frac{x\ a^2\ K\ x_E}{L^2} \qquad \text{(23)}$$

Comparing Equations (20) and (23), $\delta_E = \delta_D$, which shows that the deflection at E due to a unit load at Point D equals the deflection at Point D due to a unit load at Point E. The flexure effects for all points along the beam must be summed up to obtain the total deflections. This is not necessary to the proof, however, since, obviously, the sums of equals must be equal.

That the vertical deflection at Point D caused by a unit moment at Point A (Fig. 15(a)) is equal to the rotation at Point A caused by a unit

load at Point D (Fig. 15(b)) may be shown in a similar manner. A unit moment is applied at Point A, Fig. 15(a), and the corresponding moment diagram for the beam is shown in Fig. 15(c). Then:

$$\alpha_A = \frac{K\,a}{L} \quad \text{(24)}$$

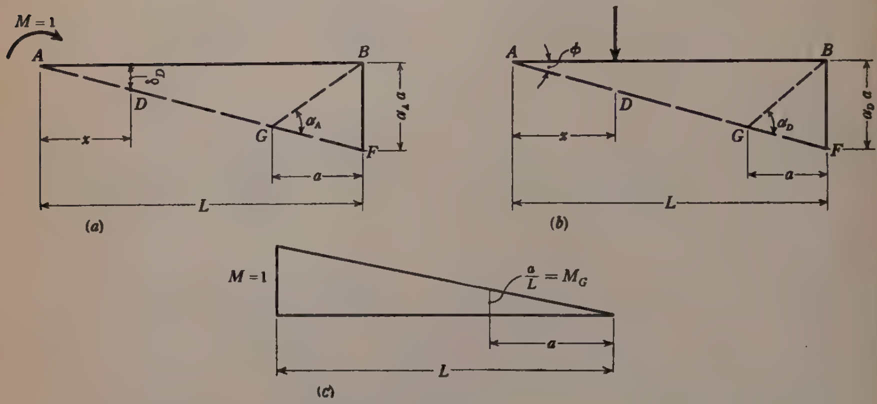

FIG. 15

and since Ordinate $BF = \alpha_A\,a$,

$$\overline{BF} = \frac{K\,a^2}{L} \quad \text{(25)}$$

The vertical deflection at Point D, Fig. 15(a), is then:

$$\delta_D = \frac{\overline{BF}\,x}{L} = \frac{K\,a^2\,x}{L^2} \quad \text{(26)}$$

The moment, M_G, at Point G (Fig. 15(b)), due to a unit load at Point D, is $\frac{x\,a}{L}$, and since $\alpha_D = K\,M_G$, it follows that:

$$\alpha_D = \frac{K\,x\,a}{L} \quad \text{(27)}$$

Again, Ordinate $BF = \alpha_D\,a$, and, therefore:

$$\overline{BF} = \frac{K\,x\,a^2}{L} \quad \text{(28)}$$

and Angle ϕ equals $\frac{\overline{BF}}{L}$, so that,

$$\phi = \frac{K\,x\,a^2}{L^2} \quad \text{(29)}$$

Comparing Equations (26) and (29), $\phi = \delta_D$, which shows that a rotation at Point A caused by a unit load at Point D equals the deflection at Point D due to a unit rotation at Point A.

L. E. GRINTER,[5] ASSOC. M. AM. SOC. C. E. (by letter).[5a]—The determination of deflections by structural geometry, as described in this paper, will serve as an excellent antidote for the attitude among undergraduate and graduate students of structures that the subject of deflections must be treated in a rather "high-brow" manner. The author has discussed a large group of problems with little more theory than that involved in a discussion of the subject of radian measure. With this paper, the terms, "structural geometry", "geometry of structures", or "geometry of deflection", "come out into the open", without becoming camouflaged by association with the dozen or more methods of deflection computation that are in use.

For some years the writer has been using the term, "structural geometry", to indicate such geometric relationships as those involved in the sketches of Fig. 16, which seem to be encompassed by the author's term, "geometry

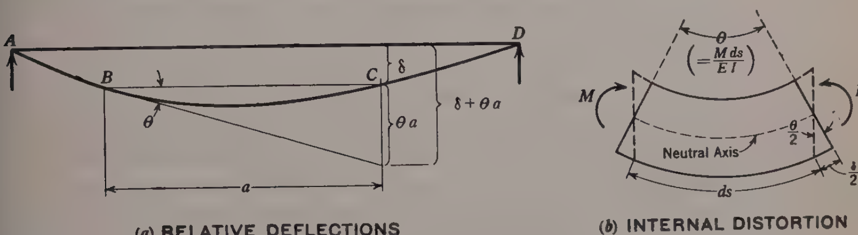

FIG. 16.—STRUCTURAL GEOMETRY.

of deflection." These relationships are so simple that they follow almost without proof and for this reason they form a proper introduction to the study of deflections. Furthermore, they involve no abstruse conceptions, such as the use of a "dummy" load, or the imaginary application of an $\frac{M}{EI}$ -loading. For the undergraduate student, or the sub-professional engineering employee, these geometric relationships are the proper starting point in a study of deflections. This, it seems, is also the author's belief. Textbooks on the strength of materials should emphasize such applications of structural geometry more decidedly than the common procedure of double integration. That authors of standard textbooks have always reversed this emphasis is one reason why undergraduate students, and even engineering graduates, often face the subject or the practice of indeterminate structural analysis and design with a complete lack of confidence.

On the other hand, there is much to be said for the methods of virtual work, area moments, the conjugate beam, elastic weights for trusses, and the Williot-Mohr diagram. Each of these methods, in certain of its forms, offers a semi-automatic procedure of deflection computation, a very desirable feature where the process is to be repeated over and over as in the analysis of certain indeterminate structures. Furthermore, the method of virtual work, for example, can be used not only to find a deflection caused by flexure, but one caused by shear, torsion, or direct stress. It is also quite as applicable

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[5a] Received by the Secretary January 6, 1937.

to the determination of rotations caused by any internal distortion. The author's conceptions of "geometry" are scarcely so extensive. For graduate study, then, assuming that the conceptions of structural geometry have been fixed in the student's mind in his undergraduate courses, the writer feels that the student should become familiar with each of the aforementioned methods. He should also learn that Castigliano's theorem is simply a reversed statement of virtual work (that is, a reversed order of the differentiation and integration), and he should be made to realize that such terms as elastic weights, moment weights, angle loads, etc., are all simply names for the same conception of an $\frac{M}{E\,I}$ - loading revised for application to trusses.

In other words, the most useful tool of deflection computation for the beginner is that method which is most nearly physical or visual; but the method that will be most useful to the experienced engineer may well be that which requires the least visualization and, therefore, tends to become most nearly automatic in its application. Here, therefore, is a conflict that will always appear. It explains why such intangible methods as virtual work and the conjugate beam are so widely useful and so widely used. Such methods are difficult to learn but easy to apply. When properly presented, the Williot-Mohr diagram can be shown to be both visual and semi-automatic. The author has presented the geometrical method as a very simple visual procedure. To interest the practical engineer, he should also emphasize its possibilities for rapid semi-automatic computations, such as those presented in Table 1.

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DISCUSSIONS

DYNAMIC DISTORTIONS IN STRUCTURES SUBJECTED TO SUDDEN EARTH SHOCK

Discussion

BY MERIT P. WHITE, JUN. AM. SOC. C. E.

MERIT P. WHITE,[12] JUN. AM. SOC. C. E. (by letter).[12a]—To engineers of certain regions the question of earthquake effects is most important. Although Professor Williams' paper does not pretend to furnish rules for the earthquake-resistant design of structures, nevertheless it does offer, to a certain extent, something which all engineers should possess before undertaking any such design, namely, an understanding of what happens to a structure subjected to an earthquake shock. The following facts (which are either stated in this paper or may be easily verified) seem especially important.

(1) Moderate viscous friction has no great effect in reducing the maximum deflections occurring in the early part of the motion (the effect of solid friction was not investigated);

(2) Amplitudes are built up very rapidly in the first stages of the motion;

(3) The region of dangerous frequencies (causing large deflections) is somewhat wider for suddenly applied simple harmonic motion than for established simple harmonic motion. (Fig. 10 shows the calculated relation between the maximum deflections of any one-story structure during one cycle or several cycles of suddenly applied simple harmonic ground motion ($Y_0 \sin F\, t$); also, for comparison, it shows the maximum deflections due to established simple harmonic motion, and $\frac{F}{F_2}$ (in distorted scale, horizontally), the ratio of the ground-motion frequency to the natural frequency of the structure);

(4) An earthquake acts not as a force, but as an irresistible motion; furthermore, the stresses produced in a structure by an earthquake are dependent not only on the ground motion (or on the maximum ground acceleration) but on the dynamical properties of the structure as well; and,

NOTE.—The paper by Harry A. Williams, Assoc. M. Am. Soc. C. E., was published in May, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1936, by Messrs. Arthur C. Ruge, and H. M. Engle.

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[12a] Received by the Secretary January 13, 1936.

(5) It can be shown that for a given amplitude and number of cycles of suddenly applied simple harmonic ground motion the maximum deflection of a structure such as the one considered herein depends only on the so-called resonance ratio, $\frac{F}{F_2}$ (see Fig. 10).

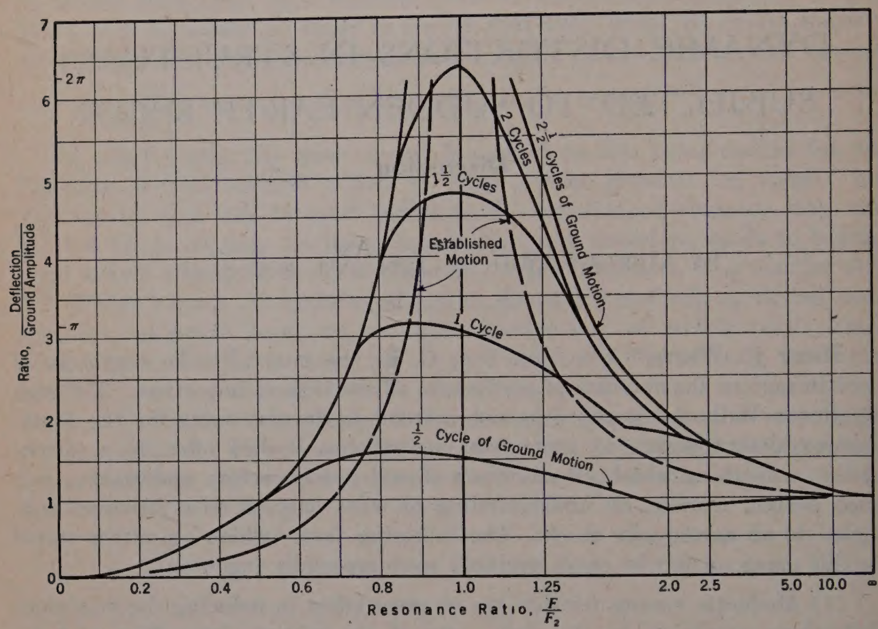

FIG. 10.—MAXIMUM DEFLECTIONS OF A SYSTEM OF ONE DEGREE OF FREEDOM, SUBJECTED TO SUDDENLY APPLIED SIMPLE HARMONIC DISPLACEMENTS AND DEFLECTIONS OCCURRING IN THE CASE OF ESTABLISHED SIMPLE HARMONIC MOTION.

From Item (5) it follows that an attempt to strengthen such a structure will affect its maximum deflections (and, therefore, its maximum stresses) only indirectly, through changing the stiffness and thus the ratio, $\frac{F}{F_2}$. This change is quite likely to increase deflections instead of decreasing them. An increase will result if $\frac{F}{F_2}$ approaches unity.

In view of the care which was probably taken in constructing the shaking-table and elevated tank model used in the author's experiments, it is not surprising that the model results should agree so well with those obtained from theory. It would be very surprising to the writer if this were not the case.

In discussing the effect of small waves preceding a major shock the author makes a relatively simple matter unnecessarily complicated. Whatever the preliminary waves and whatever the major shock, the effect of each alone as a function of time may be computed. Then, these deflections can be simply

superimposed to give the actual deflection due to the combination. In any case the preliminary waves (ceasing at the beginning of the major shock) will cause an oscillation of frequency, F_2, and of amplitude, D_0, the latter (in the notation of the paper) being expressed as:

$$D_0 = \sqrt{Y^2_0 + \left(\frac{V_0}{F_2}\right)^2} \qquad (9)$$

This oscillation can increase or decrease the maximum amplitudes which would result from the major shock alone by amounts not greater than D_0. What the effect will be, will depend on the phase difference between maxima of the motion due to the preliminary shock and those due to the major shock.

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DISCUSSIONS

INTERACTION BETWEEN RIB AND SUPERSTRUCTURE IN CONCRETE ARCH BRIDGES

Discussion

BY A. FLORIS, ESQ.

A. FLORIS,[23] ESQ. (by letter).[23a]—In his broad discussion of the interaction between arch and superstructure in concrete bridges, the author fails to include the arch-barrel type, often used in practice. The rib being a special case of the barrel, it is obvious that a general theory on the subject should start from this point. The analysis is greatly facilitated by the fact that the statically indeterminate quantities of the arch barrel as well as the superstructure in space, can be divided into two independent planar groups.

In the first group, the unknowns are influenced by moments and forces acting in the longitudinal plane of symmetry, whereas, in the second group, they are functions of moments and forces acting outside this plane. The proof of this theorem which reduces the space problem into a planar one has been given by Fr. Engesser[24] and later utilized by W. Schachenmeier in the analysis of arch bridges in which the action of the superstructure is taken into account[25]. In this elaborate analysis (the first of its kind as far as the writer knows), Schachenmeier investigates the complicated structure by choosing as a base system the fixed arch rib without deck. The analysis is complete and not difficult to apply, although the numerical work (as would be reasonable to expect) is considerable.

The aim of this discussion is merely to call attention to these two important contributions pertaining to the analysis of arched bridges. It does not constitute a criticism of the author's meritorious efforts to clarify the interaction between rib and superstructure in arch bridges.

NOTE.—The paper by Nathan M. Newmark, Jun. Am. Soc. C. E., was published in September, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

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[23a] Received by the Secretary November 18, 1936.

[24] "Das elastische Tönnengewölbe als räumliches System betrachtet," von Fr. Engesser. *Zeitschrift für Bauwesen*, 1909, pp. 107–118.

[25] "Ueber mehrfache elastische Gewölbe," von Wilhelm Schachenmeier, Leipzig, 1910.